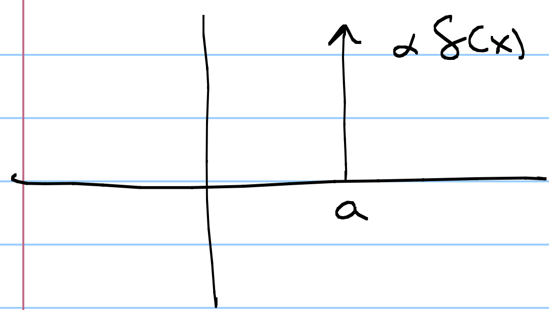
Note Title 2/27/2008

Problem 2.54

Compute the M-matrix for a single δ scatterer



for $x < a$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

For $x > a$ $\quad Fe^{ikx} + Ge^{-ikx}$

so continuity of ψ at a yields

$$\psi(a_-) = \psi(a_+)$$

$$Ae^{ika} + Be^{-ika} = Fe^{ika} + Ge^{-ika}$$

$$\Rightarrow \boxed{Ae^{2ika} + B = Fe^{2ika} + G}$$

$$\left.\frac{\partial\psi}{\partial x}\right|_{x=a_-} - \left.\frac{\partial\psi}{\partial x}\right|_{x=a_+} = \text{jump in } \psi'$$

$$ikAe^{ika} - ikBe^{-ika} - ikFe^{ika} + ikGe^{-ika}$$

recall jump in $\frac{d\psi}{dx} \Rightarrow + \frac{2m\alpha}{\hbar^2}\psi(a)$
(2/13/08)

$$\Rightarrow ik\left[Ae^{ika} - Be^{-ika} - Fe^{ika} + Ge^{-ika}\right] = \frac{2m\alpha}{\hbar^2}\left[Ae^{ika} + Be^{-ika}\right]$$

$$\Rightarrow Fe^{ika} - Ge^{-ika} = Ae^{ika} - Be^{-ika} + \frac{2i}{k}\frac{m\alpha}{\hbar^2}\left[\qquad\right]$$

$$\Rightarrow \quad Fe^{2ika} - G = Ae^{2ika} - B + i\frac{2m\alpha}{\hbar^2 k}\left(Ae^{2ika} + B\right)$$

$$Ae^{2ika} + B = Fe^{2ika} + G$$

want to combine these in the form

$$\begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

Let $\frac{m\alpha}{\hbar^2 k} = \beta$

$$\underbrace{\begin{bmatrix} e^{2ika} & -1 \\ e^{2ika} & 1 \end{bmatrix}} \begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} e^{2ika}(1+i2\beta) & -1+i2\beta \\ e^{2ika} & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

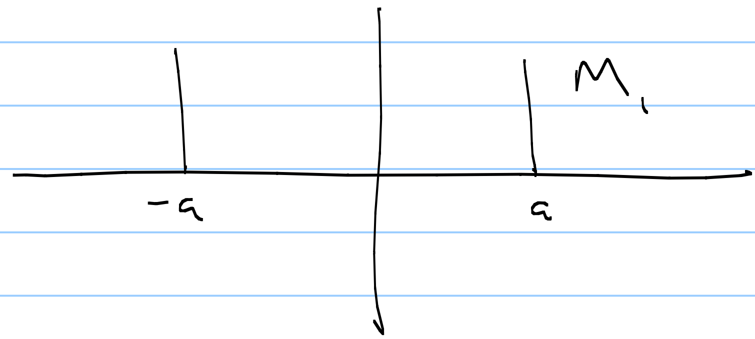
inverse =

$$\frac{1}{2}\begin{bmatrix} e^{-2ika} & e^{-2ika} \\ -1 & 1 \end{bmatrix}$$

matrix multiply gives

$\Rightarrow$

$$M = \begin{bmatrix} (1+i\beta) & i\beta e^{-2ika} \\ -i\beta e^{2ika} & (1-i\beta) \end{bmatrix} = M_1$$

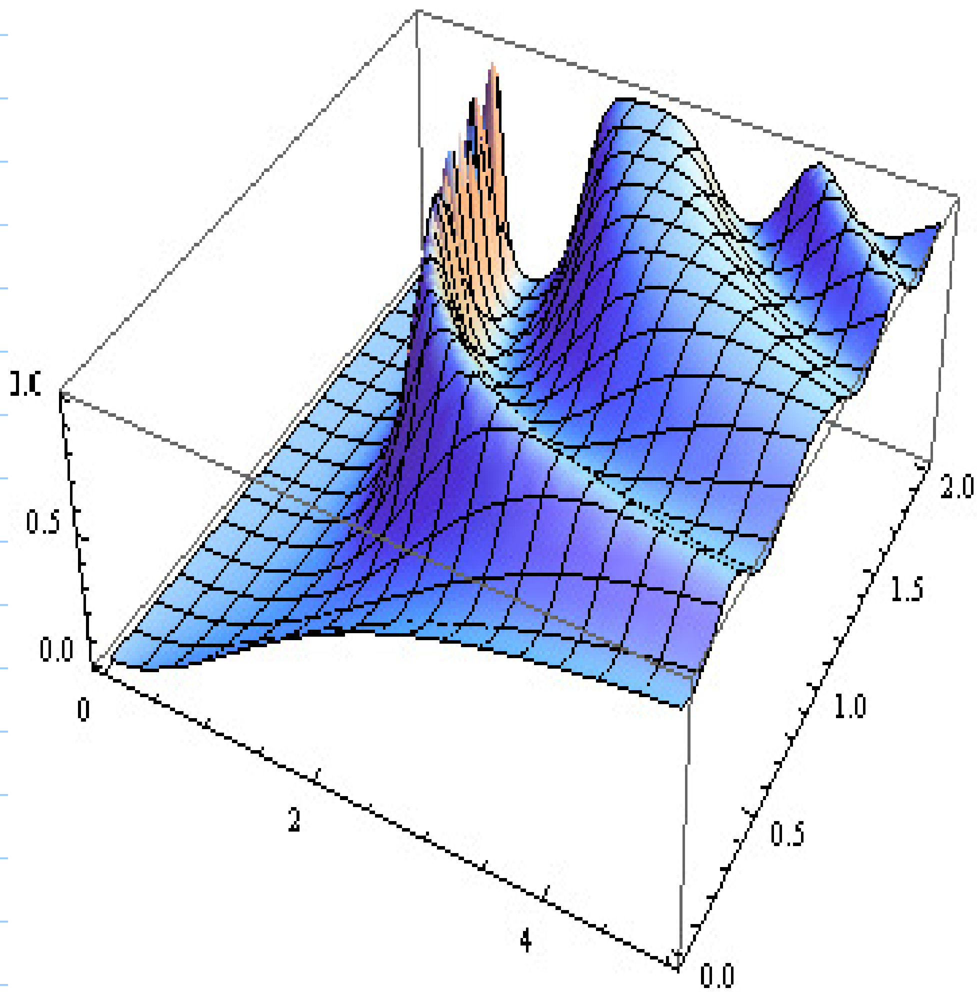


to get M_2 $\quad a \Rightarrow -a$

$$M_2 = \begin{pmatrix} (1+i\beta) & i\beta e^{2ika} \\ -i\beta e^{-2ika} & (1-i\beta) \end{pmatrix}$$

Then $M = M_2 M_1$ $\qquad$ do this

$$T = \frac{1}{|M_{22}|^2} = \frac{1}{1+4\beta^2(\cos(2ka)+\beta\sin(2ka))^2}$$



1.0
0.5
0.0
0
2
4
0.0
0.5
1.0
1.5
2.0