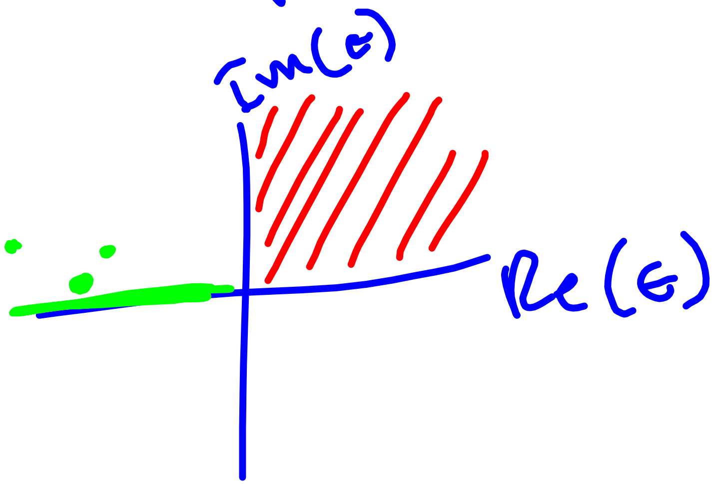
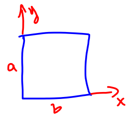
Today: Review
(some of 9.5 we didn't get to, a little more)

Conductivity, perfect conductors, and "perfect metals"

$$k^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$$



Wave guides



The book does TE case

Let's do TM case

$$E_z = X(x)\,Y(y)$$

$$\nabla_T^2 E_z + \Big(\underbrace{\frac{\omega^2}{v^2}}_{k^2} - k_z^2\Big) E_z = \emptyset$$

$$\frac{\partial^2}{\partial x^2}XY + \frac{\partial^2}{\partial y^2}XY + \left(\frac{\omega^2}{v^2} - k_z^2\right)XY = \emptyset$$

Aside: Wave eqn: $\nabla^2 f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t} = \emptyset$

Assume harmonic t dep.

$$\nabla^2 f + \frac{\omega^2}{c^2} f = 0$$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right) f = \emptyset$$

$$(\nabla^2 + k^2) f = \emptyset$$

Helmholtz Eqn.

$$\frac{1}{X}\frac{\partial^2}{\partial x^2}X + \frac{1}{Y}\frac{\partial^2}{\partial y^2}Y + \left(\frac{\omega^2}{c^2} - k_z^2\right) = \emptyset$$

$$\frac{1}{X}\frac{\partial^2}{\partial x^2}X = C_x \qquad \frac{1}{Y}\frac{\partial^2}{\partial y^2}Y = C_y$$

$$\underbrace{C_x}_{-k_x^2} + \underbrace{C_y}_{-k_y^2} + \left(\frac{\omega^2}{c^2} - k_z^2\right) = \emptyset$$

$$\Rightarrow \frac{\omega^2}{c^2} - k_z^2 - k_x^2 - k_y^2 = \emptyset$$

Can't satisfy BC's.

$C_x > 0$ $\quad X(x) = A e^{+\sqrt{C_x}\,x} + B e^{-\sqrt{C_x}\,x}$

$C_x < 0$ $\quad X(x) = \cancel{A}\cos(k_x x) + B\sin(k_x x)$

$-k_x^2 = C_x$ $\quad \emptyset$ because $X(0) = \emptyset$

$C_y < \emptyset$ $\quad Y(y) = C\sin(k_y y)$ $\quad$ { cos term also violates BC's.

$$X(x)\Big|_{x=b} = \emptyset = B\sin(k_x b)$$

$$k_x b = n\pi \Rightarrow k_x = \frac{n\pi}{b}$$

Similarly, $k_y = \frac{m\pi}{a}$

$$E_z(x,y) = C_1 \sin(k_x x)\sin(k_y y)$$

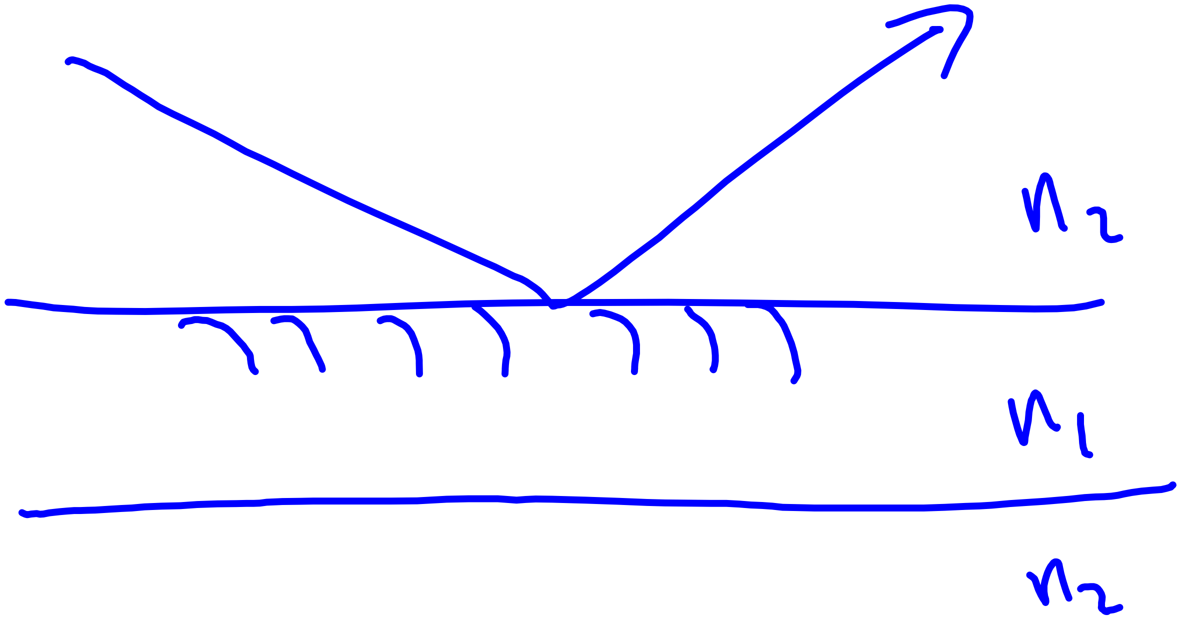
$$k_x = n\pi/b,\ k_y = m\pi/a$$

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{v^2}$$

$$\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 + k_z^2 = \frac{\omega^2}{v^2}$$

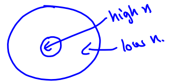
Our original assumption was

$$\vec{E} = \vec{E}_0(x,y)\, e^{i(k_z z - \omega t)}$$

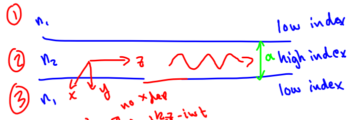


n2
n1
n2

Dielectric Guiding
like in fiber optic cable



1-D case



no x dep

$$\vec{E} = \vec{E}_0(\cancel{x}, y)\, e^{ik_z z - i\omega t}$$

For guiding as $y \to \infty, -\infty$ $\vec{E}$ must $\to \emptyset$

Assume no x dependence.

Let's treat TE case $\Rightarrow \vec{E}_0 = E_0(x)\hat{x}$

$$\nabla^2 \vec{E} + \frac{\omega^2}{v^2}\vec{E} = \emptyset \qquad \begin{cases} \nabla_T^2 E_z + \left(\frac{\omega^2}{v^2} - k_z^2\right) E_z = \emptyset \\ \nabla_T^2 E_x + \left(\frac{\omega^2}{v^2} - k_z^2\right) E_x = \emptyset \end{cases}$$

$$\left(\underbrace{\cancel{\frac{\partial^2}{\partial x^2}}}_{\to \emptyset} + \frac{\partial^2}{\partial y^2}\right) E_x + \left(\frac{\omega^2}{v^2} - k_z^2\right) E_x = \emptyset$$

$$\frac{\partial^2 E_x}{\partial y^2} + \left(\frac{\omega^2}{v^2} - k_z^2\right) E_x = \emptyset$$

$$\frac{1}{E_x}\frac{\partial^2 E_x}{\partial y^2} + \left(\frac{\omega^2}{v^2} - k_z^2\right) = \emptyset$$

$$\frac{1}{E_x}\frac{\partial^2 E_x}{\partial y^2} = C \qquad = -\left(\frac{\omega^2}{v^2} - k_z^2\right)$$

$C > 0$: $E_x(y) = A e^{+\sqrt{C}\,y} + B e^{-\sqrt{C}\,y}$

$\sqrt{C} = \alpha \to C = \alpha^2$

$C < 0$: $-k_y^2 = C$

$E_x(y) = D\cos(k_y y) + F\sin(k_y y)$

Region 1: $A e^{\alpha y}$ $\quad \frac{\omega^2}{v^2} - k_z^2 + \alpha^2 = \underbrace{\frac{\omega^2 n_1^2}{c^2} - k_z^2}_{<\emptyset} + \alpha^2 = \emptyset$

Region 2: even: $D\cos(k_y y)$ $\quad \underbrace{\frac{\omega^2 n_2^2}{c^2} - k_z^2}_{>\emptyset} - k_y^2 = \emptyset$

odd: $F\sin(k_y y)$

Region 3: $B e^{-\alpha y}$

even $B = A$, odd $B = -A$.

$E_{\parallel}$ is continuous accrosss all boundaries.

$\Rightarrow A e^{-\alpha a/2} = D\cos\left(-k_y \frac{a}{2}\right) = D\cos\left(k_y \frac{a}{2}\right)$

$\frac{\partial E(y)}{\partial y}$ is also continuous. $\left(\frac{1}{\mu_1} = \frac{1}{\mu_2}\right)$.

$\Rightarrow A\alpha e^{-\alpha a/2} = D k_y \cos\left(-k_y \frac{a}{2}\right)$

Even case

$$\boxed{\begin{aligned} &A e^{-\alpha a/2} = D\cos(k_y a/2) \\ &A\alpha e^{-\alpha a/2} = D k_y \cos\left(k_y \frac{a}{2}\right) \\ &\frac{\omega^2 n_1^2}{c^2} - k_z^2 + \alpha^2 = \emptyset \\ &\frac{\omega^2 n_2^2}{c^2} - k_z^2 - k_y^2 = \emptyset \end{aligned}}$$