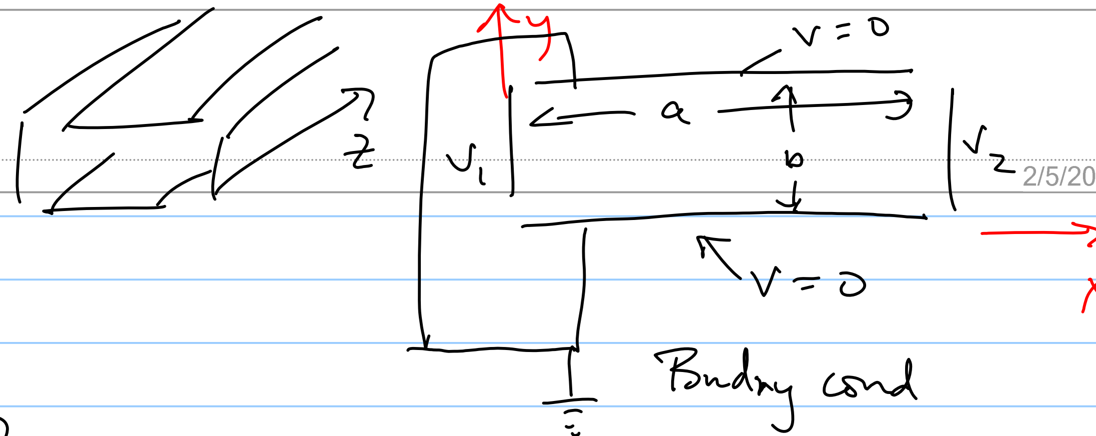
Sep. Variables



$$V(x,y,z) = X(x)\, Y(y)\, Z(z) \qquad 3\text{-ODE}$$

$$Y = A \sin ky + B \cos ky \qquad c_2 < 0$$

$$Y = A' e^{\sqrt{c_2}\, y} + B' e^{-\sqrt{c_2}\, y} \qquad c_2 > 0$$

$$Y = A'' + B'' y \qquad c_2 = 0$$

$$\nabla^2 V = 0 = \underbrace{\frac{1}{X}\frac{d^2 X}{dx^2}}_{c_1} + \underbrace{\frac{1}{Y}\frac{d^2 Y}{dy^2}}_{c_2} + \underbrace{\frac{1}{Z}\frac{d^2 Z}{dz^2}}_{c_3}$$

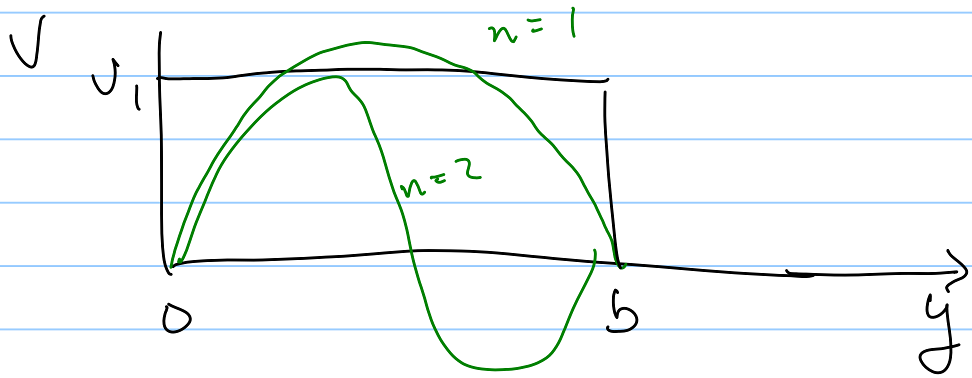
$$0 = c_1 + c_2 + c_3$$

$$\boxed{Y(y) = A \sin\left(\frac{n\pi}{b} y\right)}$$

$$X = G e^{kx} + H e^{-kx}$$

$$V(x, y, z) = \left(G e^{kx} + H e^{-kx}\right) \sin ky$$

$$k = \frac{n\pi}{b}$$



at $x = 0$ bndry !

If PDE is linear $\sum_{j=1}^{\infty} V_j$ is also a soln

$$V(x,y,z) = \sum_{n=1}^{\infty} \left(A_n e^{-\frac{n\pi x}{b}} + B_n e^{n\pi x/b} \right) \sin\left(\frac{n\pi y}{b}\right)$$

determine const by bndry cond'n

at $x=0$ $\quad V(0) = V_1 = \sum \left(A_n + B_n \right) \sin\left(\frac{n\pi y}{b}\right)$

multiply by $\sin \frac{m\pi y}{b}$ & integrate $0 \to b$

$$\underline{V_1} \int_0^b \sin\left(\frac{m\pi y}{b}\right) dy = \sum_n \left(A_n + B_n\right) \underbrace{\int_0^b \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{m\pi y}{b}\right) dy}_{\frac{b}{2}\delta_{nm}}$$

$$V_1 \left[-\frac{b}{m\pi} \cos\left(\frac{m\pi y}{b}\right) \right]_0^b = \sum_n \left(A_n + B_n\right) \frac{b}{2} \delta_{nm}$$

$$\boxed{\left.\begin{array}{ll} m = \text{odd} & \dfrac{2bV_1}{m\pi} \\ m \text{ even} & 0 \end{array}\right\} = (A_m + B_m)\, b/2}$$

$$\Rightarrow \quad \boxed{A_n + B_n = \begin{cases} \dfrac{4V_1}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}}$$

1 eqn in 2 unks

Other bndry: $V(x=a, y) = \sum_n \left(A_n e^{-\frac{n\pi a}{b}} + B_n e^{\frac{n\pi a}{b}} \right) \sin\left(\frac{n\pi y}{b}\right)$

multiply $\sin\left(\frac{m\pi y}{b}\right)$ & integrate $0 \to b$

$$\Rightarrow \quad \boxed{A_n e^{-n\pi a/b} + B_n e^{n\pi a/b} = \begin{cases} \dfrac{4V_2}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}}$$

Solve for A_n & B_n in two boxed eqns

$$A_n = \frac{4}{n\pi}\left(\frac{V_1 - V_2 e^{-n\pi a/b}}{1 - e^{-2n\pi a/b}}\right) \qquad A_n = -B_n$$

$$V(x,y,z) = \sum_{n=\text{odd}}^{\infty} \left(A_n e^{-\frac{n\pi x}{b}} + B_n e^{\frac{n\pi x}{b}}\right) \sin\left(\frac{n\pi y}{b}\right)$$

Derive the solution to Laplace's equation in cylindrical coordinates if there is no Φ or z dependence.

$$\underbrace{\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right)}_{c_1} + \underbrace{\frac{1}{s^2}\frac{\partial V}{\partial \varphi^2}}_{c_2} + \underbrace{\frac{\partial^2 V}{\partial z^2}}_{c_3} = 0$$

$$c_1 + c_2 + c_3 = 0$$

$s \Leftrightarrow r$

V is a function of s only $\Rightarrow \frac{\partial V}{\partial \varphi} = 0 = \frac{\partial V}{\partial z} \Rightarrow c_2 = c_3 = 0$

$$\frac{1}{s}\frac{d}{ds}\left(\underbrace{s\frac{dV(s)}{ds}}_{\text{const}}\right) = 0$$

$c_1 + c_2 + c_3 = 0$

$\Rightarrow c_1 = 0$

$$\cancel{s}\,\frac{dV}{ds} = \frac{\text{const}}{s} \qquad \int dV = \int \text{const}\,\frac{ds}{s} \qquad V = A\ln s + B$$

If $V(r,\varphi) = R(r)\,\Phi(\varphi)\,\underbrace{Z(z)}_{=\ \text{const}}$ then $C_3 = 0$

$C_2 \neq 0$

$$\frac{1}{s}\frac{d}{ds}\left(s\frac{dR(r)\,\Phi(\varphi)}{ds}\right) + \frac{1}{s^2}\frac{d^2}{d\varphi}\left(R(r)\,\Phi(\varphi)\right) = 0$$

$$\underbrace{\frac{s^2\,\cancel{\Phi(\varphi)}}{\cancel{\Phi}\,R}\,\frac{1}{s}\frac{d}{ds}\left(s\frac{dR}{ds}\right)}_{C_1} + \underbrace{\frac{s^2\,\cancel{R}}{s^2\,\Phi\,\cancel{R}}\,\frac{d^2\Phi(\varphi)}{d\varphi^2}}_{C_2} = 0 \qquad \frac{1}{\Phi}\frac{d^2\Phi}{d\varphi} = C_2$$

$$\frac{1}{R}\,s\,\frac{d}{ds}\left(s\frac{dR}{ds}\right) = C_1 \Rightarrow s\left(\frac{d^2R}{ds^2} + \frac{dR}{ds}\right) = C_1 R$$