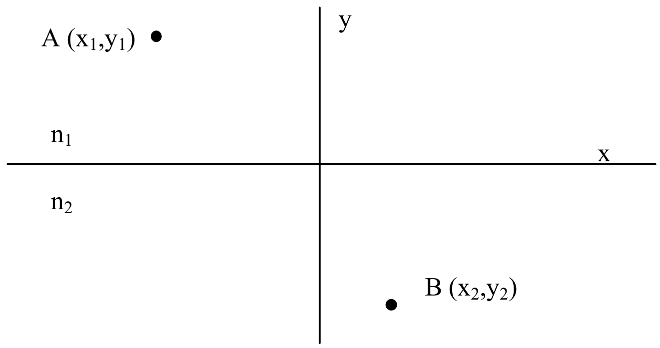
Homework 5 due 28 Sept. 2007 in class
PH462 EM Waves and Optical Physics posted: 21 September 2007

Reading: Heald and Marion (HM) chapter 6 and posted notes.

1) As discussed in class, Fermat's principle of least time states that out of all possible paths for a beam to travel between points A and B, it will take the path that minimizes the travel time. In an isotropic medium, this is just at straight line. Show that the path taken in the figure below obeys Snell's law. Hint: calculate an expression for the travel time, and determine a single parameter that you can vary to minimize the path.



2) Calculate the following quantities for an interface of vacuum with different materials: water (n=1.33), glass (n=1.5), germanium (n = 4.0):
   a. Power reflection percentage for normal incidence.
   b. Critical angle for total reflection.
   c. External (vacuum-glass) and internal (glass-vacuum) Brewster angles. Verify that light incident at the internal Brewster angle will refract at the external Brewster angle.

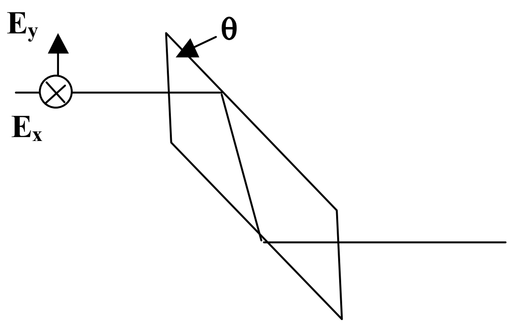
3) An isosceles dispersing prism can be made lossless for incident P-polarized light if the apex angle is made correctly. Prisms that are inside laser cavities are designed this way.
   a. Calculate an expression for the angular deviation of the refracted ray as a function of the incident angle.
   b. Show (either analytically or graphically) that when the incident angle is chosen to minimize this deviation angle, the internal ray is parallel to the base of the prism. (This is how we experimentally align the prisms.)
   c. For the 3 materials of the previous problem, calculate the apex angle for the prism such that when the beam is aligned for minimum deviation, the incident angle is at the Brewster angle.

4) A Fresnel rhomb is a prism that can be used to convert linear polarization to circular (see figure below). The input polarization is linear, with the **E**-field vector oriented at 45° to the *x*-axis (i.e. equal amplitudes for E_x and E_y). The wave enters the prism (ignore reflections at the entry point), and experiences total reflection at the two internal interfaces. Since these are parallel, the output is parallel to the input. The x- and y-components of polarization are $\perp$ and $\|$ to the plane of incidence, respectively. The angle of incidence is equal to the angle θ shown below. Each component picks up a phase shift on reflection that is calculated from the Fresnel

amplitude coefficients $r_\perp$ and $r_\parallel$. Since there is total reflection, |r| = 1, which means that we can write the amplitude coefficient as $r = e^{i\phi}$.

a. For a Fresnel rhomb in air (n = 1 outside the rhomb), and two glass refractive indices of 1.45 (quartz) and 1.65 (flint glass), use the Fresnel equation program in the posted Mathematica notebook to plot the net reflection phase shift for each polarization component vs incident angle θ. Mathematica will calculate the phase angle of a complex number z using Arg[z]. Also plot the phase difference $\Delta\phi(\theta)$ between ⊥ and || components. Note that both components pass through the same amount of glass, so we will ignore the propagation phase.

b. Determine which input angle(s), if any, can be made so that the rhomb converts the input linear polarization to circular at the output.



5) Consider reflection from a gold surface placed **in vacuum**. The real and imaginary parts of the complex refractive index ($\tilde{n} = n_R + i\,n_I$) for gold are $n_R = 1.4$, $n_I = 1.88$ at 450nm, and $n_R = 0.36$, $n_I = 10.4$ at 1500nm.. In Mathematica, use the Fresnel equation notebook posted on the course website (or your own) with these values of the complex refractive index to:

a. plot the power reflection coefficient **vs. incident angle** for both orientations and both wavelengths of the electric field, $E_\parallel$ and $E_\perp$,

b. plot the corresponding phase shifts for reflected light **vs. incident angle** for each case.