HW 2

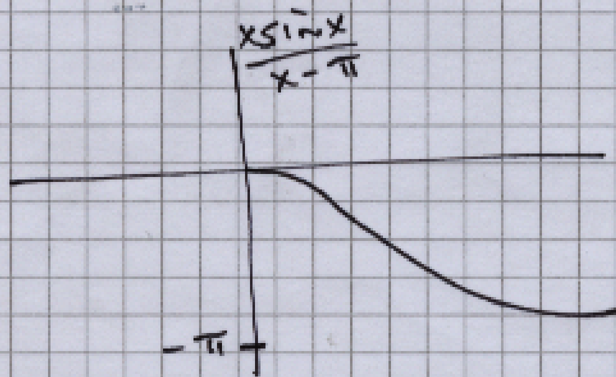
ch. 1

15.24

(a) $$\lim_{x \to \pi} \frac{x \sin x}{x - \pi}$$

$$= \lim_{x \to \pi} \frac{\sin x + x \cos x}{1}$$

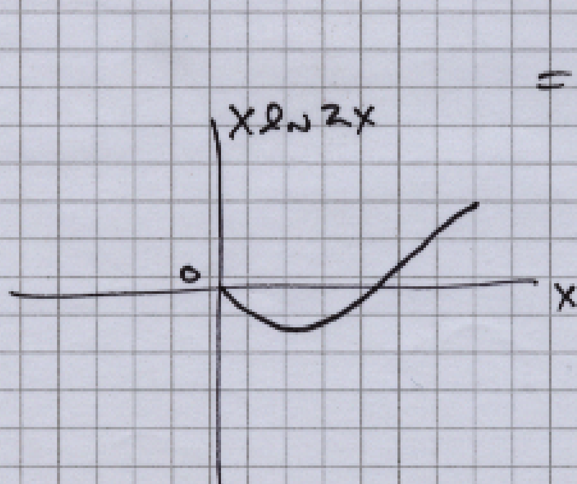
$$= \frac{0 - \pi}{1} = -\pi$$



(b) $$\lim_{x \to 0} x \ln 2x = \lim_{x \to 0} \frac{\ln 2x}{x^{-1}}$$

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x \to 0} -x = 0$$



1:15 prob. 18

$\frac{X}{19}$

$$\sum_{N=1}^{\infty} \frac{1}{N 2^N}$$ mathematica will tell you that this sums to Log(2).

guess the series arises from Log(x).

2

Now Log(x) = $-\infty$ at x=0 so consder we need to shift somehow

Log(x+1) expanded about zero is

~~Series~~ $$\text{Log}(x+1) \approx x - \frac{x^2}{2} + \frac{x^3}{3} \cdots$$

this has alternating signs, which is bad. but if we exand about x = 1 then

$$\text{Log}(x+1) \approx \text{Log}(2) + \frac{x-1}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{24} \cdots$$

these are exactly the righ coeff. except for the alt. sign but if we evaluate this at zero, then we have

$$\text{Log}(1) = \text{Log}(2) - \frac{1}{2} - \frac{1}{8} - \frac{1}{24} \cdots$$

$$0 = \Rightarrow \quad \text{Log}(2) = \frac{1}{2} + \frac{1}{8} + \frac{1}{24} \cdots$$

QED

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15 28

$$E(v) = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

$$f(v) = (1 - v^2/c^2)^{-1/2}$$

$$f(0) = 1$$

$$f'(0) = \frac{v/c^2}{(1 - v^2/c^2)^{3/2}} \Bigg|_{v=0} = 0$$

$$f''(0) = \frac{3v^2/c^4}{(\quad)^{5/2}} + \frac{1}{c^2(1 - v^2/c^2)^{3/2}} \Bigg|_{v=0}$$

$$= \frac{1}{c^2}$$

So first 2 terms $\Rightarrow$

$$f(v) = 1 + \frac{1}{2}\frac{1}{c^2}v^2$$

E(v) to the same order is

$$mc^2\left[1 + \frac{v^2}{2c^2}\right] = mc^2 + \underbrace{\tfrac{1}{2}mv^2}_{\text{class. Kinetic E.}}$$

15-30

Hw 2 Ch. 1

$$T = \frac{F}{2\sin\theta}$$

we know that $\sin\theta \simeq \theta$ for small theta, so the small angle approx to T is $\frac{F}{2\theta}$. It therefore makes sense to consider a power series in θ divided by θ so that the leading order term will be $1/\theta$

$$\frac{2T}{F} = \frac{1}{\sin\theta} = \frac{a_0 + a_1\theta + a_2\theta^2 + \cdots}{\theta}$$

a_0 should end up being 1, right?

$$\frac{\theta}{\sin\theta} = a_0 + a_1\theta + \cdots$$

$$\frac{\theta}{\sin\theta} = 1 + \theta^2/6 + \frac{7\theta^4}{360} \cdots$$

$$\Rightarrow \frac{1}{\sin\theta} \quad \frac{1}{\theta} + \frac{\theta}{6} + \frac{7\theta^3}{360} \cdots$$

$$\Rightarrow T = \frac{F}{2}\left[\frac{1}{\theta} + \frac{\theta}{6} + \frac{7\theta^3}{360} \cdots\right]$$

a) $F = 2T\sin\theta$

suppose x is small. then

$$\tan\theta = \frac{x}{10} \doteq \frac{\sin\theta}{\cos\theta} \approx \theta$$

$$x \approx 10 \cdot \theta$$

so $$T = \frac{F}{20}\left[\frac{1}{x} + \frac{x}{6} \cdots\right]$$

15:31 radius @ height $N \equiv r_N = \frac{1}{N\ln N}$

for $N \geq 2$

a) total area

$$= \pi\left[\left(\frac{1}{2\ln 2}\right)^2 + \left(\frac{1}{3\ln 3}\right)^2 + \left(\frac{1}{4\ln 4}\right)^2 \cdots\right]$$

$\ln 3 = 1.09$ so

except for the first term, each term $< \frac{1}{N^2}$ ie.

for $N = 3, 4, 5 \ldots$ $\left(\frac{1}{N\ln N}\right)^2 < \frac{1}{N^2}$

So $\underline{\underline{\text{if}}}$ $\sum_{N=2}^{\infty} \frac{1}{N^2}$ converges,

then so does $\sum_{N=2}^{\infty} \frac{1}{N \ln N}$

in fact $\sum_{N=2}^{\infty} \frac{1}{N^2} = \frac{\pi^2}{6} - 1$

digression [$\sum_{N=1}^{\infty} \frac{1}{N^2}$ is a famous series:

$H_N^{(r)} \equiv \sum_{i=1}^{N} \frac{1}{i^r}$ so our number

is related to $H_N^{(2)}$. in fact

$H_\infty^{(2)} = \frac{\pi^2}{6}$. It turns out that

$H_\infty^{(2)} = \text{Zeta}(2)$ This is the famous

Riemann zeta function

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b) Now what about the

total circumference:

$= 2\pi \sum_{N=2}^{\infty} \frac{1}{N \ln N}$ here we might

use the comp. test to compare

$\frac{1}{N \ln N} < \frac{1}{N}$ for $N \geq 3$.

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But now we're stuck with

$$\sum_{N=1}^{\infty} \frac{1}{N}$$ does this converge?

the answer can be seen by looking at large N. The sum can be approximated by an integral

$$\sum_{N=m}^{\infty} \frac{1}{N} \approx \int_m^{\infty} \frac{1}{x} dx = \ln x \Big|_m^{\infty} \Rightarrow \underline{\text{diverge}}$$

So the total circ. diverges.

c) the area decreases like $\frac{1}{r^2}$

the circum. decreases much more slowly: $\frac{1}{r}$. So slowly, in fact that the sum diverges.

Hw 2

16:18

$$\int_0^x \frac{du}{1+u^2}$$

$$\frac{1}{1+u^2} = 1 - u^2 + u^4 - u^6 \cdots$$

$$\int_0^x \frac{1}{1+u^2}\,du = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \cdots$$

$$\mathrm{ArcTan}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} \cdots$$

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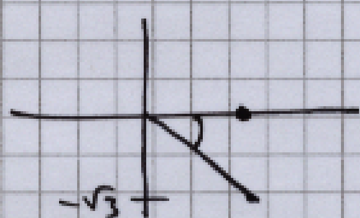
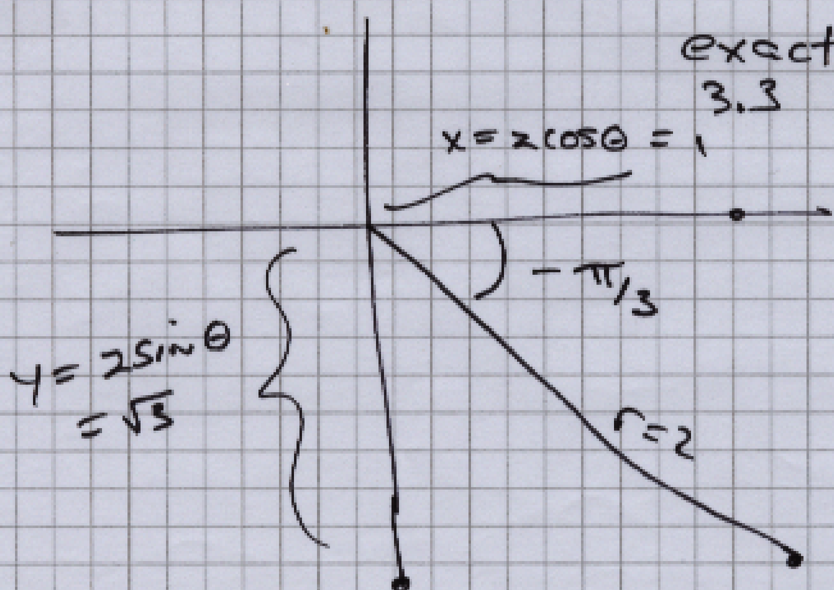
ch. 2 Sec 4 : 3

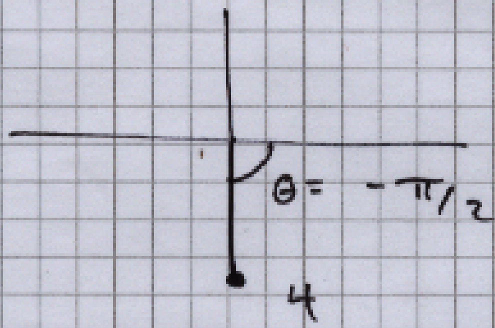
$1 - i\sqrt{3}$





exactly as in fig 3.3 excep its the complex conj.

4:6 $-4i$



x, y : $(0, -4)$

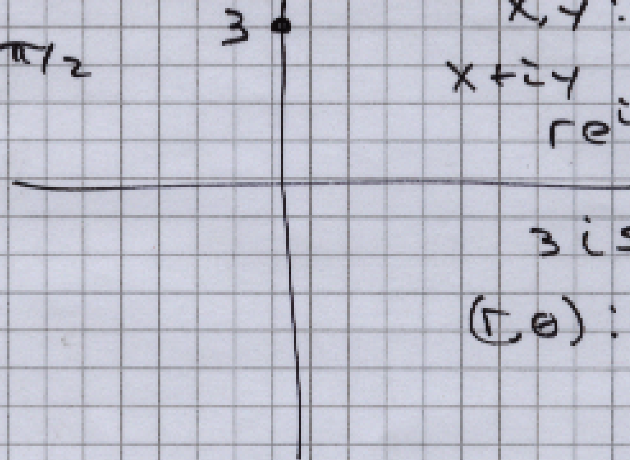
$x + iy$: $-i4$

$re^{i\theta} = 4e^{-i\pi/2}$

$4(0 - i\sin\pi/2)$

(r, θ) : $(4, -\pi/2)$

4:18 $3e^{i\pi/2}$



x, y : $(0, 3)$

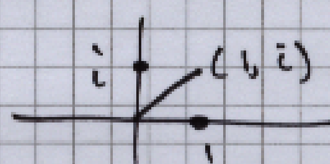
$x + iy$ $i3$

$re^{i\theta} = 3e^{i\pi/2}$

$3i\sin\pi/2$

(r, θ) : $(3, \pi/2)$

5:1 $\frac{1}{1+i}$



$\sqrt{2}\,e^{i\pi/4}$

$$\frac{1}{\sqrt{2}\,e^{i\pi/4}} = \frac{1}{\sqrt{2}}e^{-i\pi/4}$$

as a check (not required)

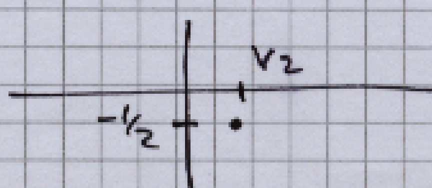
$$\frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{2}$$

$$= \frac{1}{2} - i/2$$

$x = 1/2$ $r = \frac{1}{\sqrt{2}}$

$y = -1/2$ $\theta = \tan^{-1}(-1)$

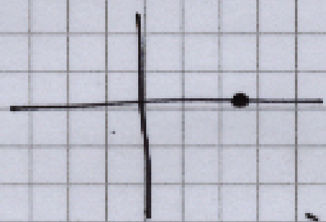
$= -\pi/4$



ch 2

5:3 $\quad i^4 = i^2 \cdot i^2$

$$= (-1)\cdot(-1) = 1$$



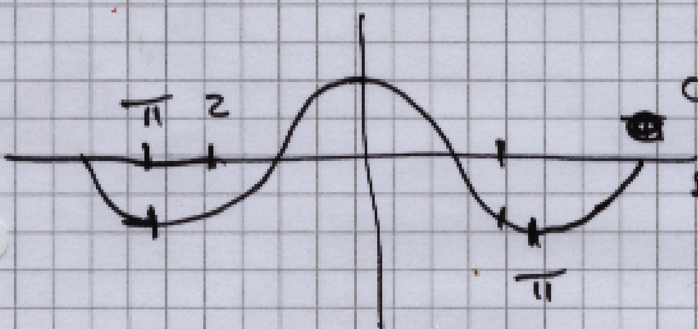
$\left.\begin{matrix} x=1 \\ y=0 \end{matrix}\right\}$ or $1e^{i0}$

NB: $\quad i = e^{i\pi/2}$

$$\Rightarrow \quad i^4 = \left(e^{i\pi/2}\right)^4 = e^{i2\pi} = e^{i0} = 1$$

9 $\quad 25e^{2i} = 25\left[\cos(2) + i\sin(2)\right]$

$$= 25\left[\underbrace{\tfrac{1}{2} + i\ .85}_{\text{rough approx.}}\right]$$



$\cos(2) \approx \frac{1}{2}$

$\sin(2) \approx \sqrt{1-1/4} \approx .85$

26 $\quad \left|\frac{2i-1}{i-2}\right| = \frac{|2i-1|}{|i-2|} = \frac{\sqrt{5}}{\sqrt{5}} = 1$

34 $\quad \frac{(i+1)^5}{(1-i)} = \frac{|i+1|^5}{|1-i|^5} = \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^5$

$$= 1$$