PHGN361 Exam 3: NAME KEY

1. Write an **integral expression** for the force on the right half of a uniformly charged sphere rotating at constant angular speed centered at and rotating about the z axis in a magnetic field $B_0\hat{z}$. For full credit be as explicit as possible (unit vectors, limits, etc.).

$d\vec{F} = \vec{J}\times\vec{B}\,d\tau \qquad \vec{J} = \rho\vec{v} \qquad \vec{v} = \vec{\omega}\times\vec{r} \qquad \vec{\omega} = \omega_0\hat{z} \qquad \vec{r} = r\hat{r} = r(\sin\theta\cos\phi\,\hat{x} + \cdots)$

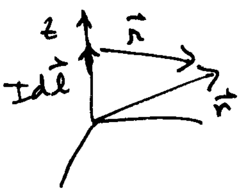
OR $\vec{v} = \omega r\sin\theta\,\hat{\phi} = \omega r\sin\theta(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})$. PUT INTO DETERMINANT

$$\vec{F} = \int d\vec{F} = \int_0^R \int^{\pi} \int^{\pi} \rho\omega r\sin\theta\left[\hat{\phi}\times B_0\hat{z}\right] r^2\sin\theta\,d\theta\,d\phi\,dr \qquad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \vec{\omega} & \cdots & \\ \vec{r} & \cdots & \end{vmatrix}$$

another determinant expressed in terms of $\hat{x}, \hat{y}, \hat{z}$ since these can be taken outside integral

See lecture 24 on wiki

2. Write an **integral expression** for the magnetic field from an infinite wire carrying constant current I_0 up the z axis using the law of Biot and Savart.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\vec{\mathscr{r}}}{\mathscr{r}^3} \qquad d\vec{l} = dz'\hat{z} \qquad \vec{r}' = z'\hat{z} \qquad \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' = x\hat{x} + y\hat{y} + (z-z')\hat{z} \qquad |\vec{\mathscr{r}}| = \sqrt{x^2+y^2+(z-z')^2}$$

$$\vec{B} = \int d\vec{B} = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi}\frac{I\,dz'}{\left[x^2+y^2+(z-z')^2\right]^{3/2}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ x & y & (z-z') \end{vmatrix}$$

See lecture 25 on wiki

3. Using Ampere's law derive the magnetic field from an infinite sheet of constant current. For full credit be as explicit as possible.

See lecture 27 on wiki

4. What current density produces constant azimuthal vector potential $A_\phi = A_0$, $A_s = 0$, and $A_z = 0$ in magnetostatics. Please put your work on the back page.