Birefringent phase matching

It is clear from the solutions of the coupled wave eqns. that high efficiency requires

$$\Delta k = k_1 + k_2 - k_3 = 0$$

When $\Delta k \neq 0$ the yield varies as

$$\mathrm{sinc}^2(\Delta k L/2)$$

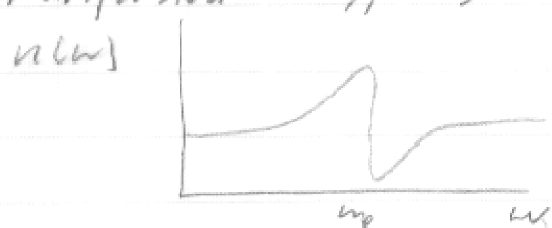
$\Delta k = 0$ sets a condition on the refr. index:

$$\Delta k = k_1 + k_2 - k_3 = n_1 \omega_1 + n_2 \omega_2 - n_3 \omega_3$$

as shown in book (2.7.7) this cannot take place with "normal" dispersion, where $n_3 > n_1, n_2$

techniques:

- anomalous dispersion: typ $\omega_3$ on high side of resonance.




examples: mixing in gas (VUV)

HHG

- birefringent PM


neg. uniaxial

> angle-tune

> temperature tune.

- Quasiphasematching: periodic modulation at $\Delta k \neq 0$
  → slow buildup in signal

Wave propagation in birefringent media.

(non magnetic, no free charge)

$$\nabla \cdot \vec{D} = 0 \qquad \nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad \nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$$

all fields will share same time dependence $\sim e^{-i\omega t}$

$$\frac{\partial}{\partial t} \to -i\omega$$

there will be a plane wave solution $\sim e^{i\vec{k}\cdot\vec{r}}$

- don't know $\vec{k}$ but can solve for it

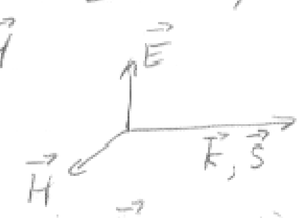
$$\vec{\nabla} \to i\vec{k}$$

$$\to \quad \vec{k}\times\vec{E} = \frac{\omega}{c}\vec{H} \qquad \boxed{\vec{k}\times\vec{H} = -\frac{\omega}{c}\vec{D}}$$

in an <u>isotropic</u> medium, $\vec{D} = \epsilon\vec{E}$ and

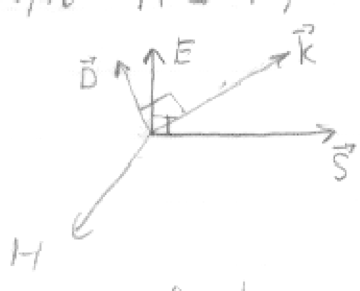
$\vec{H} \perp \vec{k}, \vec{E}$ $\qquad$ $\vec{E} \perp \vec{k}, \vec{H}$

$\therefore$ $\vec{k} \perp \vec{E} \perp \vec{H}$




$$\boxed{\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H}}$$ is in $\vec{k}$ direction

Anisotropic: $\vec{H} \perp \vec{k}, \vec{E}$ still $\vec{D} \perp \vec{H}$ also, but $\vec{D} \nparallel \vec{E}$




wavefronts are $\perp \vec{k}$

power flows along $\vec{S}$ : like "ray" in geom. optics

Birefringent case : most common by far.

3 options $\omega_3$ is along lowest index direction.

Type I:

$\omega_1$ and $\omega_2$ share same polarization.

$$\vec{E}_1 \parallel \vec{E}_2$$

Type II:

$$\vec{E}_1 \perp \vec{E}_2$$

"Type III" also called $90°$ Phase matching.

- like Type I, but input at $\theta = 90°$ and temperature tuned.
- $LiNiO_3$ most common.

Angle tuning:

use $n_e(\theta)$ function

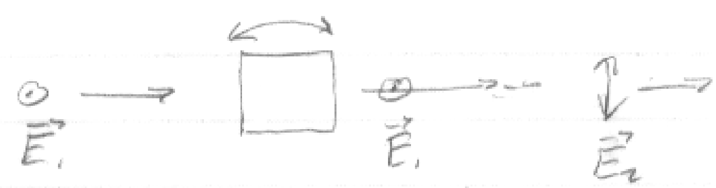
$$\frac{1}{n_e(\theta)^2} = \frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2}$$

Ex Type I: neg. uniaxial

$$n_e(2\omega, \theta) = n_o(\omega)$$

input $\vec{E}_1$ along $n_o$, vary crystal angle around $\hat{E}_1$ to tune index of $\omega_2$

$$\vec{E}_2 \perp \vec{E}_1$$




practical concerns:

- cut crystal for the process
  - x-ray diff to get angle close
- protective A/R coating or in cell to keep off water.

Anisotropic media: birefringence.

in an isotropic medium:

$$\vec{D} = \varepsilon \vec{E}$$

if linear, $\varepsilon$ doesn't depend on $\vec{E}$

$\vec{D} \parallel \vec{E}$

in an anisotropic medium, $\vec{D} \nparallel \vec{E}$

$$\vec{D} = \underset{\text{applied}}{\vec{E}} + 4\pi \underset{\text{induced polarization}}{\vec{P}}$$

b/c of asymmetry of crystal, $\vec{P} \nparallel \vec{E}$ e.g. 


write $\vec{D} = \overleftrightarrow{\varepsilon} \cdot \vec{E}$

$\overleftrightarrow{\varepsilon}$ = dielectric tensor = $\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}$ from thermodynamics, $\overleftrightarrow{\varepsilon}$ is <u>symmetric</u> $\varepsilon_{ij} = \varepsilon_{ji}$

We can choose the orientation of coordinates to diagonalize

$$\rightarrow \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \varepsilon_x & & \\ & \varepsilon_y & \\ & & \varepsilon_z \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

this coord. system is aligned along the crystal axes.

so if $\vec{E} = E_x(x,y,z)\,\hat{x}$,

$\vec{D} = P_x \hat{x} = \varepsilon_x E_x$ only

biaxial crystal: $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$

uniaxial crystal: one pair is the same. e.g. $\varepsilon_x = \varepsilon_y \neq \varepsilon_z$

isotropic crystal $\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon$ $\quad \vec{D} = \varepsilon \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \vec{E}$

The index ellipsoid.

electric energy density:

$$2U_E = \frac{\vec{D}\cdot\vec{E}}{4\pi} \quad \text{with} \quad D_x = \varepsilon_x E_x,\ D_y = \varepsilon_y E_y;\ D_z = \varepsilon_z E_z$$

in the crystal axis coord. system.

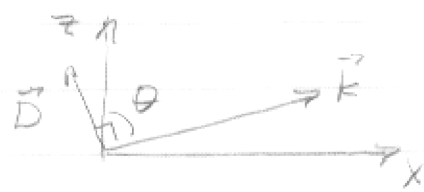
$$\rightarrow 8\pi U_E = \frac{D_x^2}{\varepsilon_x} + \frac{D_y^2}{\varepsilon_y} + \frac{D_z^2}{\varepsilon_z}$$

$$\frac{1}{8\pi U_E}\left(\frac{D_x^2}{n_x^2} + \frac{D_y^2}{n_y^2} + \frac{D_z^2}{n_z^2}\right) = 1$$

let $\hat{d} = \vec{D}/\sqrt{8\pi U_E}$ be a unit vector for the direction of $\vec{D}$

then $$\frac{\hat{d}_x^2}{n_x^2} + \frac{\hat{d}_y^2}{n_y^2} + \frac{\hat{d}_z^2}{n_z^2} = 1 \quad \text{eqn. of ellipse.}$$

Normally, we specify $\hat{k} = \dfrac{\vec{k}}{k_0}$




if $\vec{k}, \vec{D}$ are in x-z plane,

$n_x = n_o \quad n_z = n_e$

$\vec{k} = \hat{x}\, k \sin\theta + \hat{z}\, k\cos\theta$

with $k = n_e(\theta)\, k_o$

since $\vec{D} = -\dfrac{1}{k_o}\, \vec{k}\times\vec{H}$ and $\vec{H} = \hat{y} H_o$

$$= -\frac{1}{k_o}\left(-\hat{x}\, k_z H_o + \hat{z}\, k_x H_o\right)$$

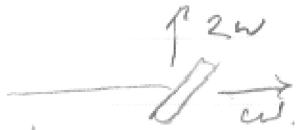
$$D_x = \frac{k_z}{k_o} H_o = n_e(\theta)\cos\theta\, H_o$$

$$D_z = (k_x/k_o)\, H_o = n_e(\theta)\sin\theta\, H_o$$

$$\rightarrow \frac{H_o^2/8\pi}{U_E}\, n_e^2(\theta)\left(\frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2}\right) = 1$$

1 equal energy $U_E = U_H$ (need proof)

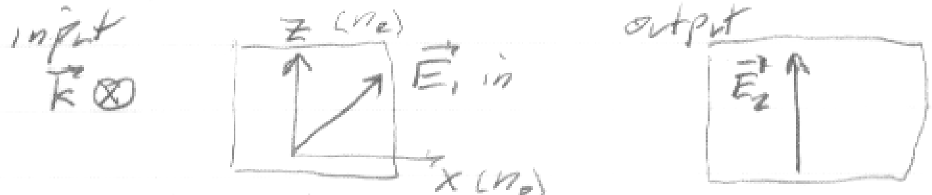
- wavelength separation:
  - prism: no background , - polarizer (too expensive)
  - dichroic mirror: typ. refl harmonic at "S"
    transm fundam. at "P"
    for best efficiency




often need multiple mirrors to get rid of fundam.

- temperature stabilization.
  - heating to drive off moisture.
  - harmonic is often partly absorbed in crystal
    $\rightarrow \Delta T \rightarrow \Delta k$
    $\therefore$ stabilize at $T > T_{room}$.
  - can fine tune $\Delta k$ with $\Delta T$

Type II:
- often seen with KTP higher nonlinearity.
- tripling after type I doubling.
- equal $\vec{E}_1$ along $n_e, n_o$.

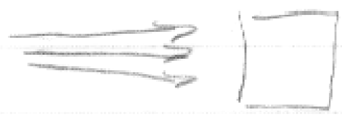



$\therefore$ output at 45° to input.
mirrors out of plane used to flip polarization.
(typically)

Angular acceptance:

input beam must be collimated.

divergent beam has a spread of input directions:



doesn't matter much in one direction

in other → conversion varies across beam

- see line go across beam.

to calculate:

plot $\text{sinc}^2(\Delta k(\theta_{opt}+d\theta)L/2)$ vs $d\theta$

some crystals more forgiving

- Type III is best.

Requires ~ several mrad, may restrict crystal length

PM bandwidth

- short pulses → range $\Delta\omega$
- plot $\text{sinc}^2(\Delta k(\omega_0+\Delta\omega)L/2)$ vs $\Delta\omega$)

at input

want > 90% across input BW

- restricts length for short pulses.

Group velocity walk off.

- connected to PM bandwidth

group delay $\tau_g = L/v_g$

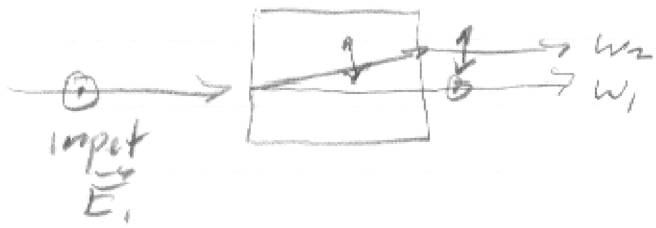
if $|\tau_g(\omega) - \tau_g(2\omega)| > \tau_p$

→ inefficient doubling.

Beam walkoff.

harmonic propagates w/ $\vec{k}$ ∦ crystal axis,

→ redirection of power flow




if using small beams (e.g. in focus)

→ power walks off, less efficient

Walkoff

consider plane wave normally incident on surface

- input $\vec{E}$ along $\vec{S}$ outside crystal
- input is at angle to crystal axes:

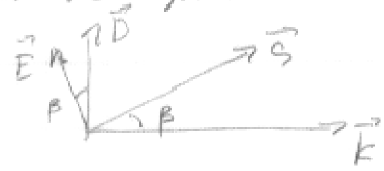

boundary conditions: $D^{\perp}$ and $E^{\parallel}$ are continuous.

know that $\vec{E} \nparallel \vec{D}$ $\therefore$ one of them bends.

$\vec{D}_{in}$ must be $\parallel$ to $\vec{D}_{out}$

but $\vec{E}_{out} = E_{in}\hat{x}' + E_{new}\hat{z}'$ is ok.

$\therefore$ inside crystal




beam can refract even at normal inc.

but opposite polarization does not refract if it's along crystal y axis

$\rightarrow$ double image for unpolarized light

two rays walk off from each other: angle $\beta$

$$\cos\beta = \frac{\vec{E}\cdot\vec{D}}{E_0 D_0} = \frac{D_x^2/\varepsilon_x + D_y^2/\varepsilon_y + D_z^2/\varepsilon_z}{E_0 D_0}$$

$$\vec{D}_{in} = D_0\hat{x}' = D_0\cos\alpha\,\hat{x} - D_0\sin\alpha\,\hat{z} \qquad D_y = 0$$

$$\cos\beta = \frac{D_0^2\left(\cos^2\alpha/\varepsilon_x + \sin^2\alpha/\varepsilon_z\right)}{D_0^2\left(\frac{\cos^2\alpha}{\varepsilon_x^2} + \frac{\sin^2\alpha}{\varepsilon_z^2}\right)^{1/2}\underbrace{\left(\cos^2\alpha + \sin^2\alpha\right)^{1/2}}_{1}}$$

calcite: $n_x = 1.658$ $\quad n_z = 1.486$ $\quad \alpha = 35°$ $\rightarrow \beta = 6.1°$

Pulse dynamics:

Ideally, pulse gets shorter during harmonic generation:

$$E_2(t) \propto E_1^2(t)$$

For gaussian input pulse,

$$E_2(t) \propto E_{10}^2 \exp\left(-2t^2/\tau_{p1}^2\right)$$

→ gaussian, with $\tau_{p2} = \tau_{p1}/\sqrt{2}$

→ corresponding changes in bandwidth.

pulse broadening:

- PM bandwidth
- GV walkoff.

Chirped input?

$|E_1(\omega)|$ unchanged, but $|E_1(t)|$ less

Nonlinearity acts in time domain!

- simulation: split-step method.

$\Delta z$ split

→ $\Delta z$ in $\omega$ domain - dispersion.

$\Delta z$ in time domain - NL