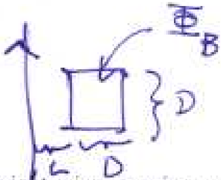


4. A square wire of side length D is in the plane with an infinite straight wire carrying current I_0. Derive an integral expression for the mutual inductance of square wire if the first side is located a distance L away from the infinite wire.

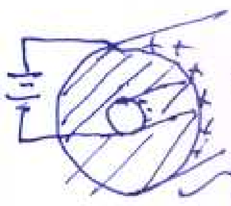
$$\boxed{\text{Defn } \Phi_B = MI}$$

mag flux

$$\Phi_B = \int \vec{B}\cdot d\vec{a} = \int_0^D \int_L^{L+D} \frac{\mu_0 I_0}{2\pi r}\hat{\varphi}\cdot dr\,dz\,\hat{\varphi}$$

Amps Law $B = \frac{\mu_0 I_0}{2\pi r}$

5. Two long metal cylinders (radii a and b) are separated by material of conductivity σ and are maintained at constant potential V_0. Starting from Ohm's law ($J = \sigma E$) derive a relationship between V_0 and I, the current between the cylinders.



$\boxed{\vec{J} = \sigma\vec{E}}$ so we need to find E. Use Gauss's Law

$$\boxed{\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}} \quad E\,2\pi r L = \frac{\lambda L}{\epsilon_0} \quad E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Fundamental Principles

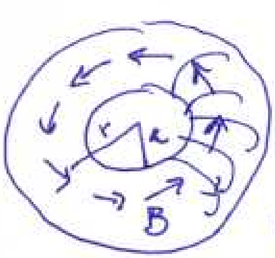
$$\boxed{I = \int \vec{J}\cdot d\vec{a}} = \int \sigma E\,da = \int_0^L \sigma \frac{\lambda}{2\pi\epsilon_0 r} 2\pi r\,dz = \boxed{\sigma\frac{\lambda L}{\epsilon_0} = I}$$

$$\boxed{\Delta V = -\int \vec{E}\cdot d\vec{l}} = \int_a^b \frac{\lambda}{2\pi\epsilon_0}\frac{dr}{r} = \boxed{\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right) = \Delta V}$$

2 eqns: eliminate λ from the two to get $\Delta V = IR$

6. Derive an expression for the magnetic field in a linear material with χ_m enclosed in a toroid carrying current I_0 with N turns wrapped per unit length with inner and outer radii a and b.

Principles $\boxed{\vec{M} = \chi_m \vec{H}}$ $\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}}$ $\Rightarrow \vec{B} = \mu_0(1+\chi_m)\vec{H}$



Need to find $\vec{H}$ use Gauss's Law $\boxed{\oint \vec{H}\cdot d\vec{l} = I_f}$

what path? B goes around toroid.

$$\oint \vec{H}\cdot d\vec{l} = H\,2\pi r = N\,2\pi a\,I_0 \qquad H = \frac{Na}{r}I_0 \;\&\; \vec{B} = \mu\vec{H}$$