Linear tensor $\chi^{(1)}$

TABLE 1.5.1 Form of the linear susceptibility tensor $\chi^{()}$ as determined by the symmetry properties of the optical medium, for each of the seven crystal classes and for isotropic materials. Each nonvanishing element is denoted by its cartesian indices

Triclinic	$\begin{bmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{bmatrix}$	
Monoclinic	$\begin{bmatrix} xx & 0 & xz \\ 0 & yy & 0 \\ zx & 0 & zz \end{bmatrix}$	
Orthorhombic	$\begin{bmatrix} xx & 0 & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{bmatrix}$	biaxial
Tetragonal Trigonal Hexagonal	$\begin{bmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{bmatrix}$	uniaxial
Cubic Isotropic	$\begin{bmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & xx \end{bmatrix}$	isotropic

NL polarization 2nd order

$$P_i^{(2)}\left(\omega_n+\omega_m\right)=\sum_{jk}\sum_{(nm)}\chi_{ijk}^{(2)}\left(\omega_n+\omega_m;\omega_n,\omega_m\right)E_j\left(\omega_n\right)E_k\left(\omega_m\right)$$

$$P_i^{(2)}\left(\omega_n+\omega_m\right)=\sum_{(nm)}\mathbf{E}\left(\omega_n\right)\bullet\overleftrightarrow{\chi}_i^{(2)}\left(\omega_n+\omega_m;\omega_n,\omega_m\right)\bullet\mathbf{E}\left(\omega_m\right)$$

$$P_1^{(2)}\left(\omega_3=\omega_1+\omega_2\right)=\begin{pmatrix} E_1\left(\omega_1\right) & E_2\left(\omega_1\right) & E_3\left(\omega_1\right)\end{pmatrix}\begin{pmatrix}\chi_{111} & \chi_{112} & \chi_{113}\\ \chi_{121} & \chi_{122} & \chi_{123}\\ \chi_{131} & \chi_{132} & \chi_{133}\end{pmatrix}\begin{pmatrix}E_1\left(\omega_2\right)\\ E_2\left(\omega_2\right)\\ E_3\left(\omega_2\right)\end{pmatrix}$$

$$+\begin{pmatrix} E_1\left(\omega_2\right) & E_2\left(\omega_2\right) & E_3\left(\omega_2\right)\end{pmatrix}\begin{pmatrix}\chi_{111} & \chi_{112} & \chi_{113}\\ \chi_{121} & \chi_{122} & \chi_{123}\\ \chi_{131} & \chi_{132} & \chi_{133}\end{pmatrix}\begin{pmatrix}E_1\left(\omega_1\right)\\ E_2\left(\omega_1\right)\\ E_3\left(\omega_1\right)\end{pmatrix}$$

Contracted notation: non-dispersive, non-absorbing medium

Non-dispersive: can permute any spatial index (Kleinmann symm)
- like colors have the same value, except black terms are unique

$$\begin{pmatrix} d_{xxx} & d_{xxy} & d_{xxz} \\ & d_{xyy} & d_{xyz} \\ & & d_{xzz} \end{pmatrix}$$

$$\begin{pmatrix} d_{xxy} & d_{xyy} & d_{xyz} \\ & d_{yyy} & d_{yyz} \\ & & d_{yzz} \end{pmatrix}$$

$$\begin{pmatrix} d_{xxz} & d_{xyz} & d_{xzz} \\ & d_{yyz} & d_{yzz} \\ & & d_{zzz} \end{pmatrix}$$

$$\begin{pmatrix} d_{xxx} & d_{xyy} & d_{xzz} & d_{xyz} & d_{xxz} & d_{xxy} \\ d_{xxy} & d_{yyy} & d_{yzz} & d_{yyz} & d_{xyz} & d_{xyy} \\ d_{xxz} & d_{yyz} & d_{zzz} & d_{yzz} & d_{xzz} & d_{xyz} \end{pmatrix}$$

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{14} & d_{12} \\ d_{15} & d_{24} & d_{zzz} & d_{23} & d_{13} & d_{14} \end{pmatrix}$$

Symmetry in d-matrices

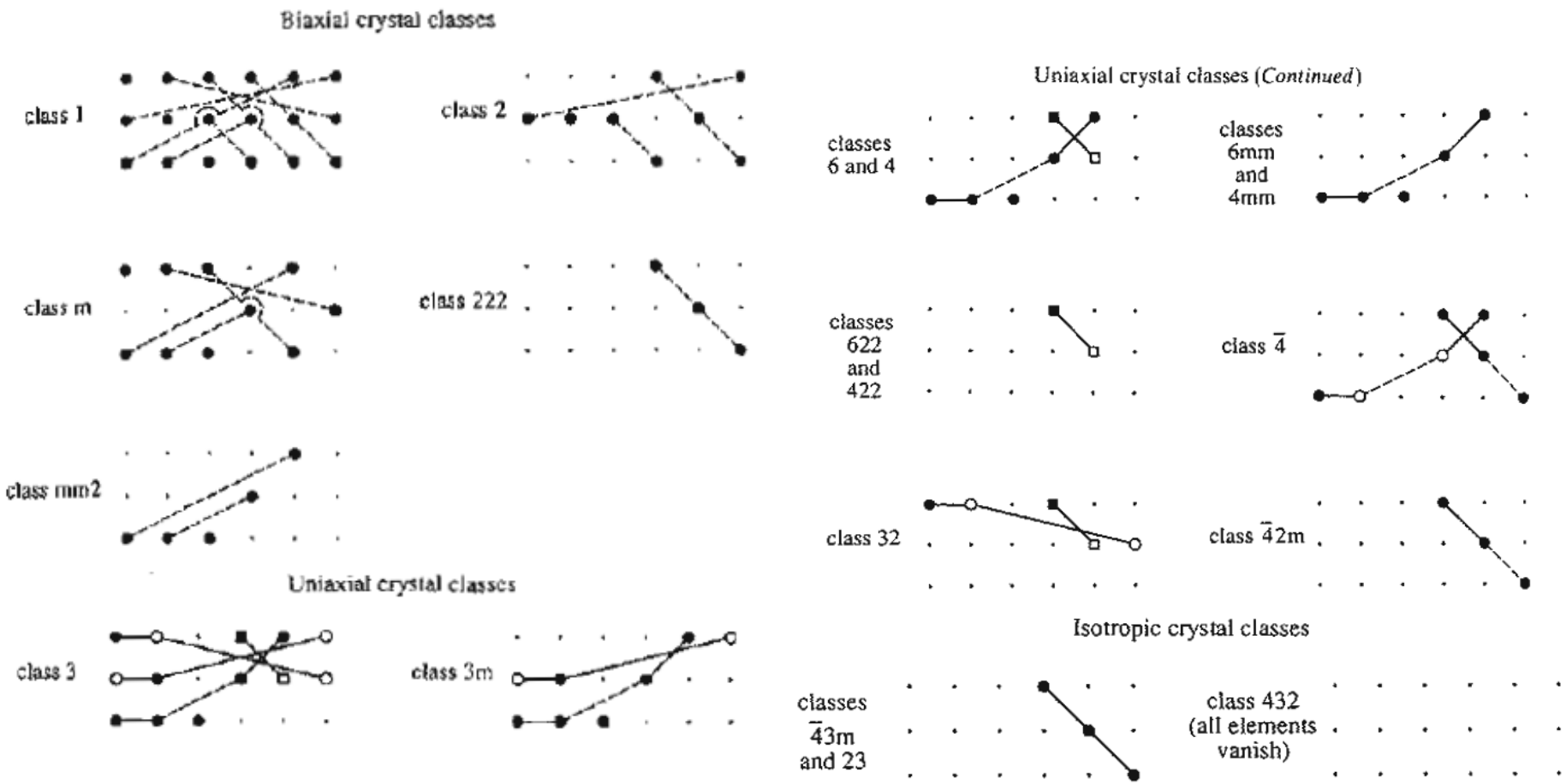

FIGURE 1.5.3 Form of the d_{il} matrix for the 21 crystal classes that lack inversion symmetry. Small dot: zero coefficient; large dot: nonzero coefficient; square: coefficient that is zero when Kleinman's symmetry condition is valid; connected symbols: numerically equal coefficients, but the open-symbol coefficient is opposite in sign to the closed symbol to which it is joined. Dashed connections are valid only under Kleinman's symmetry conditions. (After Zernike and Midwinter, 1973.)