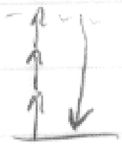
Intensity-dependent refr. index

$\chi^{(3)}$ interaction involves 4 waves FWM 4 wave mixing

THG



degenerate FWM $\rightarrow$ all waves at same ω

or

can't generate new amplitude, but waves sees NL phase shift

$$P = \underbrace{\left(\chi^{(1)} + 3\chi^{(3)}(\omega = \omega + \omega - \omega) \left| E(\omega) \right|^2 \right)}_{\chi_{eff}} E(\omega)$$

χ_{eff} connected to NL refractive index

Alert: many defn of NL index

most common: $n(I) = n_0 + n_2 I$ n_2 in cm^2/W

can express in terms of $|E|^2$, see book

Also note: effective n_2 is <u>2x higher</u> for cross modulation (>1 beam in)

Many mechanisms; diff't strength, timescales,

electronic is fastest (0.1 fs)

thermal is strong but slow.

Tensor components of $\chi^{(3)}_{ijkl}$: isotropic medium

only 4 components:

χ_{1111} ($= \chi_{2222} = \chi_{3333}$)

χ_{1122} (+ permutations of pairs e.g. χ_{2233})

χ_{1212} (permute)

χ_{1221} (")

recall classical model:

$$\vec{F} = -m\omega_0^2 \vec{r} + mb(\vec{r}\cdot\vec{r})\vec{r}$$

$$\sim -K_{eff}\vec{r} \quad \text{w/ } K_{eff} = m(\omega_0^2 - b\,\vec{r}\cdot\vec{r})$$

$\vec{F} \parallel$ to one input field

other 2 must be $\parallel$

$K_{eff} \sim m(\omega_0^2 - b)$

$\vec{F} \sim \hat{x}$

Also, $\chi_{1111} = \chi_{1122} + \chi_{1212} + \chi_{1221}$

Intrinsic permutation symm: exchange cartesian index and ω's

$\chi_{1122} = \chi_{1212} \neq \chi_{1221}$

↳ since last ω is $-\omega$

We can write full matrix as

$$\chi_{ijkl} = \underbrace{\chi_{1122}\,\delta_{ij}\delta_{kl} + \chi_{1212}\,\delta_{ik}\delta_{jl}}_{\text{equal}} + \chi_{1221}\,\delta_{il}\delta_{jk}$$

$$P_i(\omega) = 3\sum_{jkl} \chi_{ijkl}(\omega = \omega+\omega-\omega)\,E_j(\omega)\,E_k(\omega)\,E_l(-\omega)$$

$$\rightarrow 3\sum_{jkl}\left(\chi_{1122}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}\right) + \chi_{1221}\,\delta_{il}\delta_{jk}\right)E_j E_k E_l^*$$

$\rightarrow \vec{E}\cdot\vec{E}$

NL polarization rotation

When a NL material is exposed to elliptical light, the material has an induced optical activity.

we can describe any polarization state as a linear combination of
$R(\hat{\sigma}_-)$ or $L(\hat{\sigma}_+)$ circ. polarization states:

$$\vec{E} = E_+\hat{\sigma}_+ + E_-\hat{\sigma}_- \qquad \hat{\sigma}_\pm = \tfrac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y})$$

our NL polarization is

$$\vec{P}^{NL} = A(\vec{E}\cdot\vec{E}^*)\vec{E} + \tfrac{1}{2}B(\vec{E}\cdot\vec{E})\vec{E}^*$$

express both sides in terms of $\hat{\sigma}_+$, $\hat{\sigma}_-$ vectors

$$P_+\hat{\sigma}_+ + P_-\hat{\sigma}_- = A\left|E_+\hat{\sigma}_+ + E_-\hat{\sigma}_-\right|^2(E_+\hat{\sigma}_+ + E_-\hat{\sigma}_-) + \tfrac{1}{2}B(E_+\hat{\sigma}_+ + E_-\hat{\sigma}_-)^2\cdot (E_+^*\hat{\sigma}_+^* + E_-^*\hat{\sigma}_-^*)$$

$$\hat{\sigma}_\pm^* = \hat{\sigma}_\mp$$
$$\hat{\sigma}_\pm \cdot \hat{\sigma}_\mp^* =$$
$$\hat{\sigma}_\pm \cdot \hat{\sigma}_\pm^* = 1 \quad \text{(normalized)}$$

grouping terms:

$$P_+ = A(|E_+|^2 + |E_-|^2)E_+ + BE_+E_-E_-^*$$
$$= A|E_+|^2E_+ + (A+B)|E_-|^2E_+ = \left(A|E_+|^2 + (A+B)|E_-|^2\right)E_+$$
$$\equiv \chi_+^{NL}E_+$$
$$P_- = A(|E_+|^2 + |E_-|^2)E_- + BE_+E_-E_+^*$$
$$= A|E_-|^2E_+ + (A+B)|E_+|^2E_- = \left[A|E_-|^2 + (A+B)|E_+|^2\right]E_-$$
$$= \chi_-^{NL}E_-$$

so the NL interaction separates:

each R, L component exper. a different effective n's

$$n_\pm^2 = n_0^2 + 4\pi\chi_\pm^{NL}$$

$\Rightarrow$ diff't v_{ph} $\qquad \Delta n = n_+ - n_- = \dfrac{2\pi B}{n_0}\left(|E_-|^2 - |E_+|^2\right)$

this takes the form of an induced optical activity.

→ NL ellipse rotation

- linear, circular polarization are unaffected.

This effect is used for pulse contrast enhancement

- gas hollow fibers.
- BaF_2

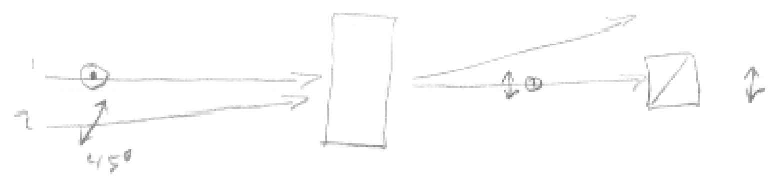
Polarization gating:

2 input beams

$\vec{E}_1 = E_1 \hat{x}$ weak

$\vec{E}_2 = E_2 \cdot \frac{1}{\sqrt{2}} (\hat{x} + \hat{y})$ strong.

$\vec{E}_1$ is rotated by $|E_2|^2$



$$E_{1y} \sim |E_2(\tau)|^2 \chi^{(3)}_{eff} E_{1x}$$

use for FROG (pulse characterization) and as an optical shutter.

B- integral

the effect of n_2 is to modify the phase of the beam

ex: self-focusing:

$$\phi^{NL}(r) = k_0 n_2 I(r) L$$

for gaussian beam

$$I(r) \sim I_0 e^{-2r^2/w^2} \sim I_0 (1 - 2r^2/w^2) \text{ near } r=0$$

$$\phi^{NL}(r) \sim k_0 n_2 I_0 L - 2k_0 n_2 I_0 r^2 L / w^2$$

paraxial spherical wave: $e^{-k_0 r^2/2f}$

$$\frac{k_0}{2f_{NL}} = \frac{2k_0 n_2 L I_0}{w^2}$$

$$f_{NL} \approx \frac{w^2}{4 n_2 I_0 L}$$

when $f_{NL} > z_R$ with $L = z_R$

$$\frac{f_{NL}}{z_R} = \frac{w^2 \pi w^2}{4 n_2 P_0 n_0} \cdot \frac{\lambda_0^2}{n_0^2 \pi^2 w^4} = \frac{\lambda_0^2}{4\pi n_0^3 n_2 P_{cr}} = 1$$

critical power

$$P_{cr} \approx \frac{\lambda_0^2}{4\pi n_0^3 n_2} \quad \left(\text{more accurate: } \frac{\pi (0.61)^2 \lambda_0^2}{8 n_0 n_2} \right)$$

(see ch 7)

A measure of whether NL phase is important is if

max $\Phi_{NL} > 1$

define $B = \frac{2\pi}{\lambda} \int n_2 I(z) \, dz$

note beam size, duration can change w/ z.

for self-focusing @ $B = 1$

$\rightarrow \quad k_0 n_2 I_0 L = 1$

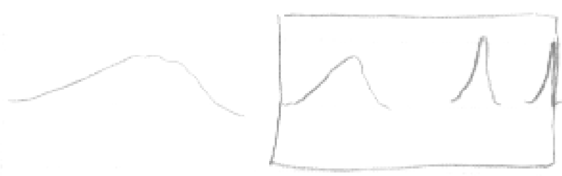
$$f_{NL} = \frac{k_0 w^2}{4} = \frac{\pi w^2}{2\lambda} \sim \frac{1}{2} z_R$$

estimate in amplifiers

$$B = \sum_n \frac{\mathcal{E}_n \cdot n_2 L_{cr}}{\pi w^2 \tau_p}$$

∴ stretch pulses to avoid NL effects.

pulse compression in glass:



$\tau_p = \tau(z)$

integrate thru crystal.