

$$(1) \nabla \times \mathbf{E}_1 = k \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3zx \end{vmatrix} = k [\hat{x}(0 - 2y) + \hat{y}(0 - 3z) + \hat{z}(0 - x)] \neq 0,$$

so \mathbf{E}_1 is an *impossible* electrostatic field.

$$(2) \nabla \times \mathbf{E}_2 = k \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + z^2 & 2yz \end{vmatrix} = k [\hat{x}(2z - 2z) + \hat{y}(0 - 0) + \hat{z}(2y - 2y)] = 0,$$

so \mathbf{E}_2 is a *possible* electrostatic field.

Let's go by the indicated path:

$$\mathbf{E} \cdot d\mathbf{l} = (y^2 dx + (2xy + z^2)dy + 2yz dz)k$$

$$\text{Step I: } y = z = 0; dy = dz = 0. \mathbf{E} \cdot d\mathbf{l} = ky^2 dx = 0.$$

$$\text{Step II: } x = x_0, y: 0 \rightarrow y_0, z = 0. dx = dz = 0.$$

$$\mathbf{E} \cdot d\mathbf{l} = k(2xy + z^2)dy = 2kx_0y dy.$$

$$\int_{II} \mathbf{E} \cdot d\mathbf{l} = 2kx_0 \int_0^{y_0} y dy = kx_0y_0^2.$$

$$\text{Step III: } x = x_0, y = y_0, z: 0 \rightarrow z_0; dx = dy = 0.$$

$$\mathbf{E} \cdot d\mathbf{l} = 2kyz dz = 2ky_0z dz.$$

$$\int_{III} \mathbf{E} \cdot d\mathbf{l} = 2y_0k \int_0^{z_0} z dz = ky_0z_0^2.$$

$$V(x_0, y_0, z_0) = - \int_0^{(x_0, y_0, z_0)} \mathbf{E} \cdot d\mathbf{l} = -k(x_0y_0^2 + y_0z_0^2), \text{ or } \boxed{V(x, y, z) = -k(xy^2 + yz^2)}.$$

$$\text{Check: } -\nabla V = k \left[\frac{\partial}{\partial x}(xy^2 + yz^2) \hat{x} + \frac{\partial}{\partial y}(xy^2 + yz^2) \hat{y} + \frac{\partial}{\partial z}(xy^2 + yz^2) \hat{z} \right] = k[y^2 \hat{x} + (2xy + z^2) \hat{y} + 2yz \hat{z}] = \mathbf{E}. \checkmark$$

