Rules: open book, open notes, open internet, no communication with others. You may want to use the fourier tranform sheets posted on the website. You may use extra pages for calculations (or mathematica), but please put your answers on these pages to make it easier to grade.

1) Light is incident on the plano-concave shape as shown below, where the radius of curvature of the curved surface is $R=3 \mathrm{~cm}$. The object is made of sodium metal, with a refractive index $n(\omega)=\sqrt{1-\omega_{p}^{2} / \omega^{2}}$, where $\omega_{p}$ is the plasma frequency. The number density of the electrons in the metal is $N_{e}=2.5 \times 10^{22} \mathrm{~cm}^{-3}$.


## Work the following questions out for two wavelengths of light:

$\lambda_{1}=500 \mathrm{~nm}$ and $\lambda_{2}=50 \mathrm{~nm}$.
a) Numerically calculate the index of refraction in the metal for this incident wavelength.

At $\lambda_{1}=500 \mathrm{~nm}, \mathrm{n}=$
At $\lambda_{2}=50 \mathrm{~nm}, \mathrm{n}=$
b) With the index of refraction written as above, is there any energy absorbed in the metal? Explain why or why not for each wavelength.
c) On the figure above, carefully draw the direction of the ray for each of the two incident wavelengths, showing the change in direction at each interface.
2) Consider Young's double slit experiment with a slit separation $a$ and finite slits of equal width $b$ (see diagram below). The illumination of the slits is uniform from the left with polarization aligned with the slits. The intensity distribution pattern is shown for the case where the slits are illuminated with polarized coherent light and $a=3 b$. The upper curve is an envelope and the lower (faster oscillating) curve is the diffraction pattern intensity. $\mathrm{x}_{1}$ is the fringe spacing and $x_{2}$ is the distance from the peak of the envelope to its first zero.

For this problem, it is sufficient to give a solid verbal explanation of your answers. You may choose to support your answers with calculation. If you get your answer by calculation I still want you to explain the result in general terms.

a) Suppose we do something to rotate the polarization of the light that passes through the lower slit by $90^{\circ}$. Sketch the observed intensity distribution under these conditions. Make clear what, if any, changes there are to $\mathrm{I}_{\mathrm{pk}}, \mathrm{x}_{1}$ and $\mathrm{x}_{2}$. Explain your answer.
b) Next we go back to the original setup then use a thin plate to introduce a $\pi$ phase shift on the light passing through one slit relative to the other. What happens to the distribution?
c) We again go back to the original setup, then narrow the slit widths by a factor of two ( $b^{\prime}=b / 2$ ) while maintaining their separation $\left(a^{\prime}=a\right)$. Aside from letting less light through, how is the pattern affected by this change in slit size? Below, express the new values $x_{1}^{\prime}$ and $x_{2}^{\prime}$ in terms of the original values.
$x_{1}^{\prime}=$
$x_{2}^{\prime}=$
Sketch the new pattern.
d) Again, back to the original slit width, then we increase the separation of the slits by a factor of 2: $a^{\prime}=2 a, b^{\prime}=b$. Sketch the new intensity pattern on the screen, and describe the changes:
$x_{1}^{\prime}=$
$x_{2}^{\prime}=$
3) A propagating mode of a planar waveguide has the electric field $E_{0} \cos \left(\frac{5 \pi x}{a}\right) \exp \left[i\left(k_{z} z-\omega t\right)\right]$, where $a$ is the full width of the waveguide (see figure below).
a) Assuming the index inside the waveguide core is $n=1.5$, the vacuum wavelength is $\lambda_{0}=1 \mu \mathrm{~m}$ and the width of the waveguide is $a=5 \mu \mathrm{~m}$, calculate the components of the wavevector:
$k_{x}=$
$k_{z}=$
b) The phase velocity for an unbounded plane wave in glass of index $n$ is $c / n$. Expressing the phase velocity of the wave as $\mathrm{v}_{p h}=\alpha c / n$, calculate $\alpha$ :
$\alpha=$
Does the phase velocity increase or decrease as the mode number increases? Explain why.
c) In the ray picture of a guided wave, there are rays reflecting from the waveguide walls as shown below. What is the angle, $\theta$, of the ray to the $z$-axis for the mode described in part a?

$\theta=$
4) A plane wave $E_{0} e^{i(k z-\omega t)}$ is normally incident (see parallel input rays in the figure below) on a fully transparent window of thickness $d$. This window is doped with a material such that its refractive index varies linearly with position with the following function:
$n(y)=n_{0}+\Delta n(y / a)$, where $\Delta n \ll n_{0}$.

a. Write an expression for the field for $\mathrm{z}>0$ that accounts for the effects of the gradientindex window. Assume that $\mathrm{z}=0$ at the exit plane of the window and ignore the edges of the window.
$E(y, z, t)=$
b. On the figure above, sketch the direction that the input rays shown in the figure after they leave the window (this may help you with your expression in part a).
c. What is another kind of optical element that has a similar effect on a beam?
d. Now an aperture of full width $2 a$ surrounds this window and a lens focuses the resulting diffraction pattern on a screen that is placed one focal length from the lens. Calculate the Fraunhofer diffraction pattern that is on the screen, including the effects of the aperture and this gradient-index window.


