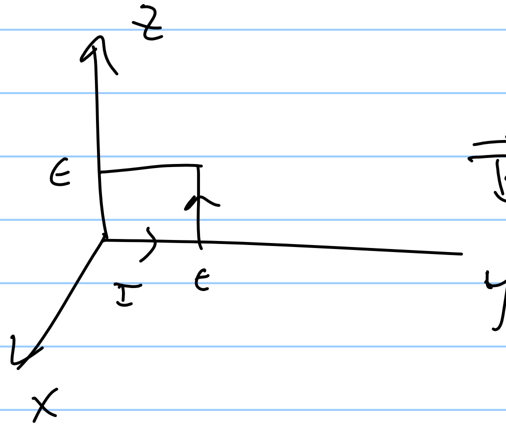
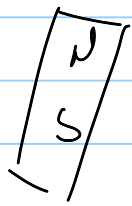
Note Title 4/27/2009



ϵ is small

$$\vec{B} = B_x(x,y,z)\hat{x} + B_y(x,y,z)\hat{y} + \qquad \hat{z}$$

4 $d\vec{\ell}$'s



$$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$$

lower, right

$$= I\Big\{ dy\,\hat{y} \times \vec{B}(0,y,0) + dz\,\hat{z} \times \vec{B}(0,\epsilon,z)$$

upper

$$- dy\,\hat{y} \times \vec{B}(0,y,\epsilon) - dz\,\hat{z} \times \vec{B}(0,0,z)\Big\}$$

+ & limits differen't

$$d\vec{F} = I\Big\{ -dy\,\hat{y} \times \underbrace{\Big[\vec{B}(0,y,\epsilon) - \vec{B}(0,y,0)\Big]}_{\epsilon \left.\frac{\partial \vec{B}}{\partial z}\right|_{0,y,0}} + dz\,\hat{z} \times \underbrace{\Big[\vec{B}(0,\epsilon,z) - \vec{B}(0,0,z)\Big]}_{\epsilon \left.\frac{\partial \vec{B}}{\partial y}\right|_{0,0,z}}$$

Taylor series

$$\vec{B}(0,\epsilon,z) = \vec{B}(0,0,z) + \epsilon \left.\frac{\partial B}{\partial y}\right|_{0,0,z} + \ldots$$

$$\int_0^\epsilon dy \left.\frac{\partial \vec{B}}{\partial z}\right|_{0,y,0} \simeq \epsilon \left.\frac{\partial \vec{B}}{\partial z}\right|_{0,0,0}$$

constant & same derivative as at 0,0,0

$$\vec{F} = I\epsilon^2 \left[\hat{z} \times \frac{\partial \vec{B}}{\partial y} - \hat{y}\frac{\partial \vec{B}}{\partial z} \right]$$

$\uparrow$ m magnetic dipole mom

$$\vec{F} = m \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ \frac{\partial B_x}{\partial y} & \frac{\partial B_y}{\partial y} & \frac{\partial B_z}{\partial y} \end{vmatrix} - m \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1 & 0 \\ \frac{\partial B_x}{\partial z} & \frac{\partial B_y}{\partial z} & \frac{\partial B_z}{\partial z} \end{vmatrix}$$

$$= m \left\{ \hat{y}\frac{\partial B_x}{\partial y} - \underline{\hat{x}\frac{\partial B_y}{\partial y} - \hat{x}\frac{\partial B_z}{\partial z}} + \hat{z}\frac{\partial B_x}{\partial z} \right\}$$

use $\vec{\nabla}\cdot\vec{B} = 0 = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \Rightarrow \frac{\partial B_x}{\partial x} = \underline{-\frac{\partial B_y}{\partial y} - \frac{\partial B_z}{\partial z}}$

$$\vec{F} = m \left\{ \hat{x}\frac{\partial B_x}{\partial x} + \hat{y}\frac{\partial B_x}{\partial y} + \hat{z}\frac{\partial B_x}{\partial z} \right\}$$

But $\vec{m}\cdot\vec{B} = m B_x$ since $\vec{m} = m\hat{x}$

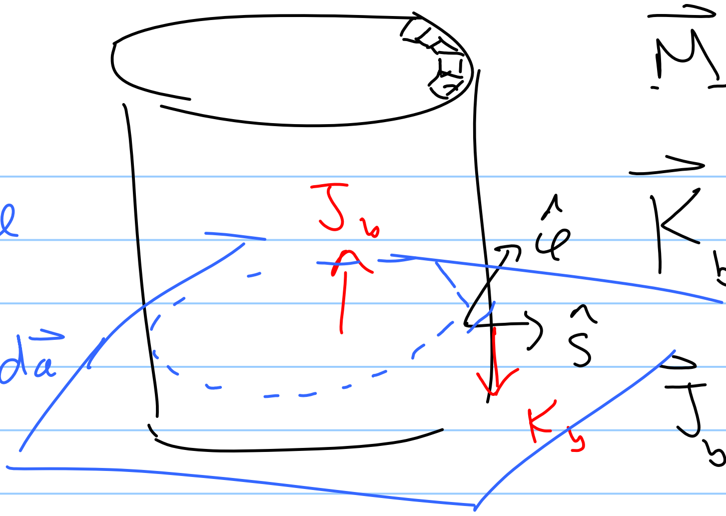
So $\vec{\nabla}(\vec{m}\cdot\vec{B}) = m\vec{\nabla}B_x = m\left(\frac{\partial B_x}{\partial x}\hat{x} + \frac{\partial B_x}{\partial y}\hat{y} + \frac{\partial B_x}{\partial z}\hat{z} \right)$

$= \vec{F}$

$$\vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B})$$

$\int K\,dl$

$I = \int \vec{J}_b \cdot d\vec{a}$



$\vec{M}$ dipole/vol

$\vec{K}_b = \vec{M} \times \hat{n}$

$\vec{J}_b = \vec{\nabla} \times \vec{M}$

$\vec{M} = k s^2 \hat{\varphi}$

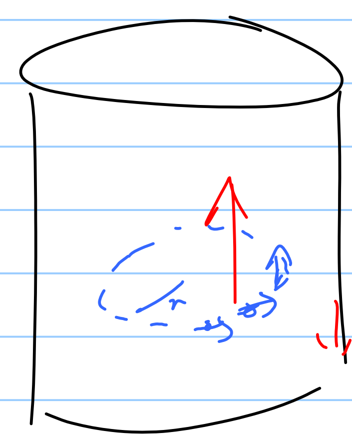
↑ radial direction

Prin: find K_b & J_b → Biot S or Amps

Meth: $K_b = \vec{M} \times \hat{n}$: $\hat{\varphi} \times \hat{s}$

$\vec{J}_b = \vec{\nabla} \times \vec{M}$

large cylind



B from Amp's Law

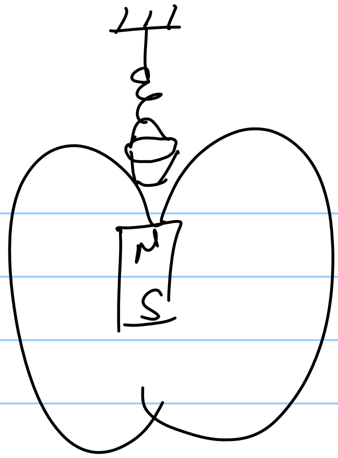
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{end}$$

$$I_{end} = \int \vec{J} \cdot d\vec{a}$$

$$\Downarrow$$

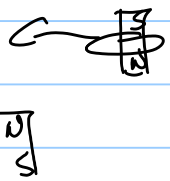
$$B\,2\pi r = \int \vec{J} \cdot d\vec{a}$$

Real materials



- strongly attracted : Ferromag
- weak attraction : paramag
- repulsion : diamag

Faradays law



mag statics

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} = \mu_0 \vec{J}_{free} + \mu_0 \vec{J}_{bound}$$

$$\vec{J}_{bound} = \vec{\nabla} \times \vec{M}, \qquad \mu_0 \vec{J}_{bound} = \mu_0 \vec{\nabla} \times \vec{M}$$

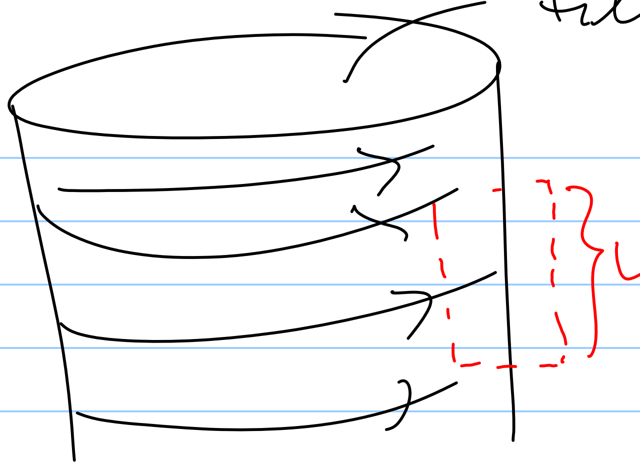
$$\vec{\nabla} \times \underbrace{\left(\frac{\vec{B}}{\mu_0} - \frac{\mu_0 \vec{M}}{\mu_0} \right)}_{\vec{H}} = \vec{J}_f$$

$$\vec{\nabla} \times \vec{H} = J_f \qquad \oint \vec{H} \cdot d\vec{l} = I_f$$

$\cdot\, da$ $\qquad \cdot\, da$

stokes Th

fill with H_2O



$$\oint \vec{H}\cdot d\vec{l} = HL$$
$$= nLI_0$$

find $H = nI_0$

Assume linear material

$$\vec{M} \propto \vec{H}$$

paramag pos
diamag neg

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_0} - \chi_m \vec{H}$$

$$\underbrace{\mu_0(1+\chi_m)}_{\mu}\,\vec{H} = \vec{B}$$

$$\mu \vec{H} = \vec{B}$$