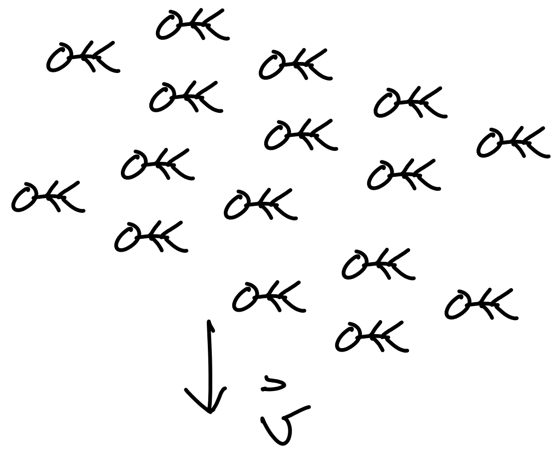
Lecture 19 Review of vector potential and Ohm's law

Muddiest points:

- What is E analogous to in the skydiver problem? What is J in the skydiver problem? Don't you need a wire with cross sectional area to define J?

$$\vec{J} = \rho \vec{v}$$

ρ: $\frac{\#\text{ charges}}{m^3}$ $\frac{m}{s}$



$$\rho = \frac{\#\text{skydivers}}{m^3}$$

v = velocity of skydivers

$$\boxed{\vec{J} = \sigma \vec{E} \qquad \vec{F} = q\vec{E} \qquad \vec{E} = \frac{\text{force}}{\text{charge}}}$$

$\vec{E}$: $\frac{\text{force}}{\text{charge}}$

Ohm's law

$$\vec{J} = \rho \vec{v} = \Sigma_1 \vec{g}$$

$\vec{g}$: $\frac{\text{force}}{\text{skydiver}}$ skydiver's law

Σ_1: constant; what does large Σ_1 mean?

For large value it means large J at fixed g. If the skydiver density does not change then the skydivers must have a large velocity. For small value their velocity is small. This could be cause by thinner air or less friction.

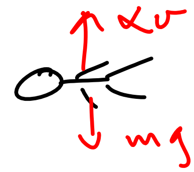
Increase E and J increases in wire. ρ_e stays same $\Rightarrow$ v increases

Increase g and J for skydivers increases. $\rho_{\text{skydivers}}$ stays same $\Rightarrow$ v increases

-Where does the work done by gravity go?

$$W_{nc} = \Delta(\cancel{KE}^{\,0} + PE) \qquad W_{per\ skydiver} = mg\Delta y$$

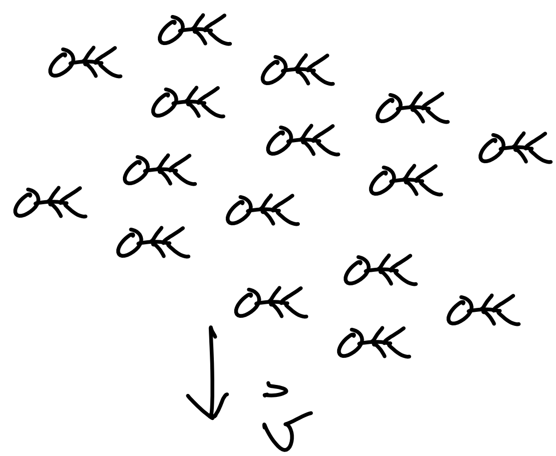
This generates heat.



$$P_{ower} = \vec{F}\cdot\vec{v} = mgv$$

$$P_{per\ diver} = \vec{F}\cdot\vec{v} = gv$$

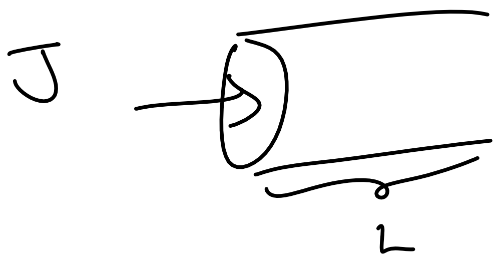
$$P_{all\ divers} = \int \underbrace{\rho\, d\tau}_{\#\ \text{of divers}}\ \overbrace{gv}^{\text{power per diver}}$$



$$P_{all\ divers} = \int \underbrace{\rho\, d\tau\ \ g v}_{J} = \int g \underset{\Sigma g}{\underset{\parallel}{J}}\, d\tau = \int \Sigma g^2\, d\tau$$

$$P_{circuit} = \int \sigma E^2\, d\tau = \int \underbrace{\sigma E}_{J}\, E\, d\tau$$

-How is this form of Ohm's law related to V=IR?



$$I = \int J\, da = JA$$

$\uparrow$ constant J

$$JA = \sigma E A$$

$$\Delta V = \left| -\int_0^L \vec{E} \cdot d\vec{r} \right| = EL \Rightarrow E = \frac{\Delta V}{L}$$

$$I = JA = \sigma \frac{\Delta V}{L} A \, ; \quad \boxed{\Delta V = \underbrace{\frac{L}{\sigma A}}_{= R} I = IR}$$

$$P_{circuit} = \int \sigma E^2 \, d\tau = \int \underbrace{\sigma E}_{J} \, \underbrace{E\, dz}_{dV} \, \underbrace{dx\, dy}_{da}$$

$$= \Delta V \int J\, da = \Delta V\, I$$

-(analogy)Does the electron have a parachute?

The microsopic model of a conduction electron is more complicated. See Shadowitz section 9.1

-(analogy)What analogy is related to the capacitor?

See the next lecture.

-(modifying) What happens to the model for large distance the skydiver falls (G vs g)?

This is like the electric field varying in the wire.

-(modifying) What happens to the model as the pressure changes with altitude?

This is like varying the conductivity.

-Go over hmwk ocean problem

$$\vec{J} = \sigma \vec{E}$$

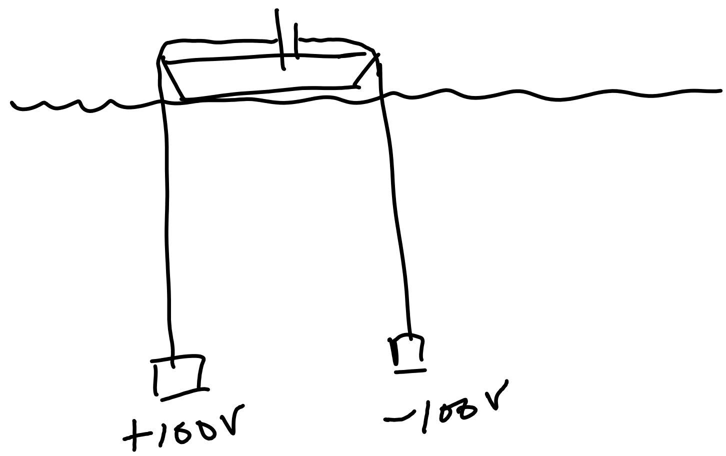
$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{J} = \nabla\cdot(\sigma E)$$

assume const

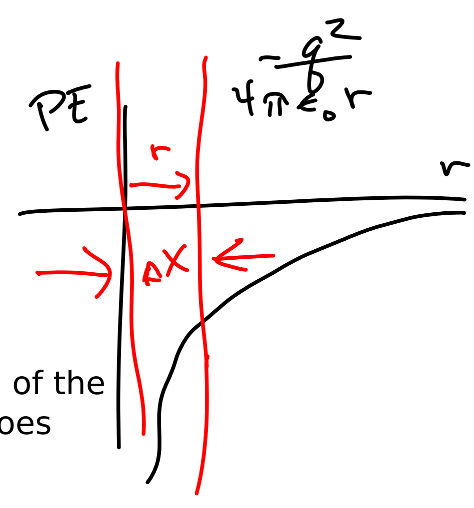
$$\sigma\, \vec{\nabla}\cdot\vec{E} = 0, \quad \vec{E} = -\vec{\nabla} V$$

$$\nabla^2 V = 0$$



-hydrogen atom: why was r replaced by Δx

This is a model of the energy in the atom. We let the width of the well be associated with the radius. Obviously the PE well does not look much like an infinite well. However, it is easy to calculate the kinetic energy in this model well.



- Fill in the lecture notes so they are more like a book.

I was thinking of having the students fill out the lecture notes and hand it in as part of the homework assignment. You should be looking at these notes and letting me know what is confusing on InkSurvey.

-Lectures are scattered. Make them focussed.

I need more information than such a general comment.

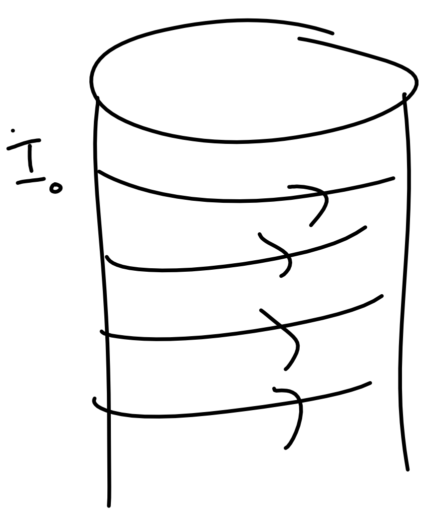
-Do separation of variables and not other methods.

We will get to separation of variables. It is good to see other methods because separation of variables is not always the easiest way to a soln.

-Stop going over PH200 material.

-I can't answer all questions so email (if you think I'm too grumpy) or see me.

-more examples of the vector potential.



What is the direction of A?

$$A_x(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J_x(\vec{r}')\,d\tau'}{|\vec{r}-\vec{r}'|}$$

assuming ~~V~~ J_x goes to zero at infinity

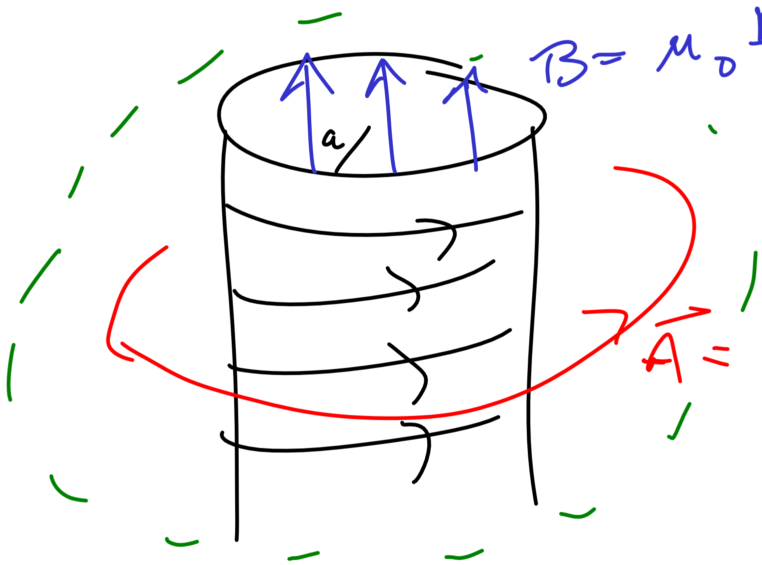
$$A_y = \int J_y$$

$$A_z = \int J_z$$

$n\ z$

surface current

$$\vec{B} = \mu_0 \frac{N}{L} I_0 = \mu_0 K$$



$$\vec{A} = A_\varphi \hat{\varphi}$$

$$\Phi_B = \int \vec{B}\cdot d\vec{a} = \underbrace{\int \vec{\nabla}\times\vec{A}\cdot da}_{\text{Stokes theorem}}$$

From Ampere's Law

$$\int \mu_0 K\, da$$

$$\mu_0 K \pi a^2$$

$$\oint \vec{A}\cdot d\vec{r}$$

$$= A\, 2\pi r$$

$$\vec{A} = \frac{\mu_0 K \pi a^2}{2\pi r} = \frac{\mu_0 K a^2}{2r}\hat{\varphi}$$

Is B zero outside the solenoid?

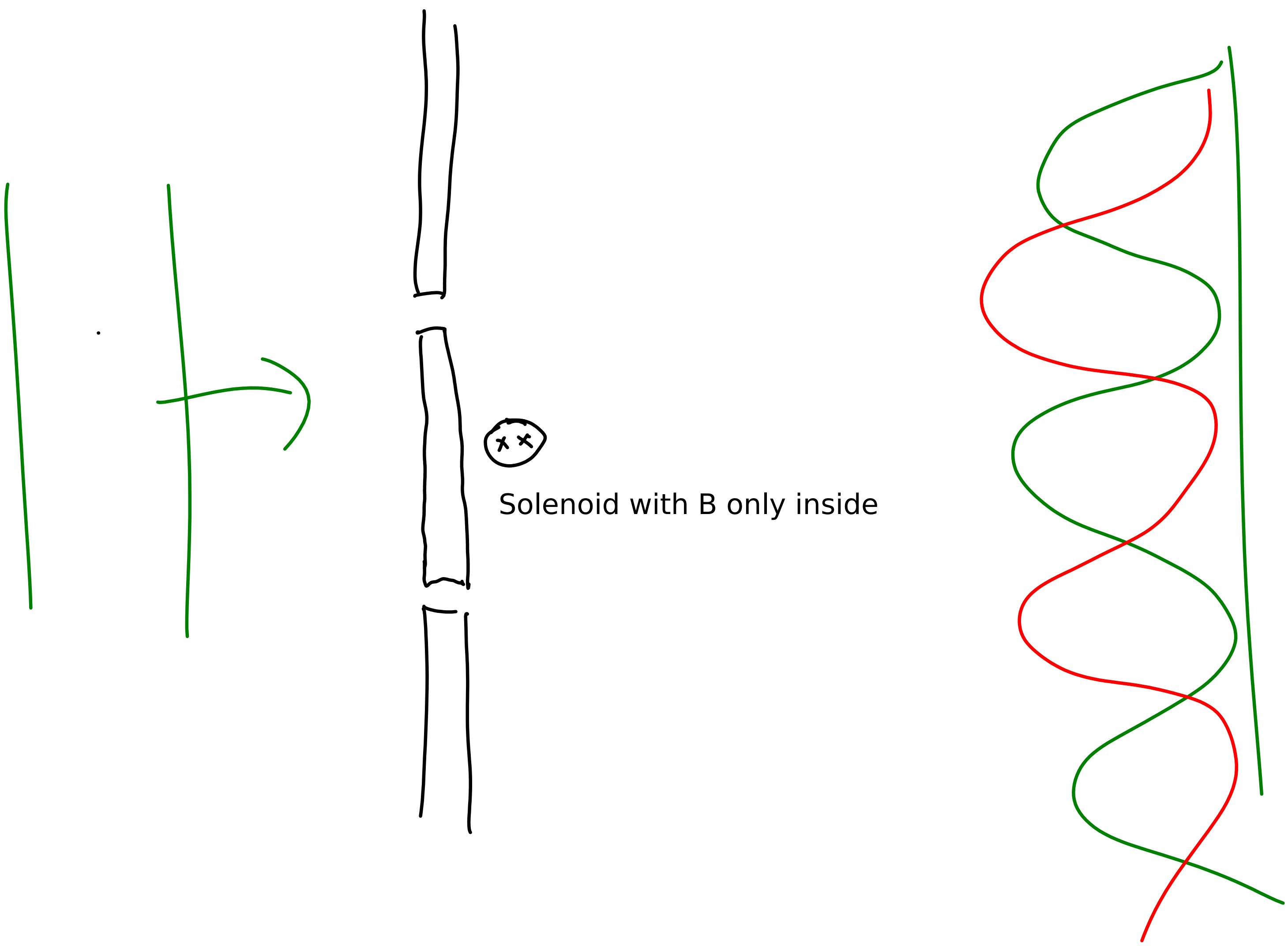
YES.

Is A zero outside the solenoid?

NO.

(causal-creative) How would you test how A appears in QM?

QM differs from classical mechanics because the motion of the particle can be determine by a wave. The example most used is the double slit expt.



Review of the analogy between V and A.

$$\nabla^2 V = -\rho/\epsilon_0$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(x',y',z')}{r}\,dx'dy'dz'$$ assuming V goes to zero at infinity

vector Laplacian

$$\vec{\nabla}^2\vec{A} = -\mu_0\vec{J} \rightarrow$$

$$\hat{x}\,\nabla^2 A_x + \hat{y}\,\nabla^2 A_y + \hat{z}\,\nabla^2 A_z = -\mu_0\left(J_x\hat{x} + J_y\hat{y} + J_z\hat{z}\right)$$

or 3 eqns

$$\nabla^2 A_x = -\mu_0 J_x\,;\quad \nabla^2 A_y = -\mu_0 J_y\,;\quad \nabla^2 A_z = -\mu_0 J_z$$

The similarity of the A partial diff. eqns and the voltage partial diff. eqn. suggests that we use a similar integral solution.

By analogy

$$\begin{cases} A_x(\vec{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \dfrac{J_x(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|} \\ A_y = \displaystyle\int J_y \\ A_z = \displaystyle\int J_z \end{cases}$$ assuming V goes to zero at infinity

n z

Note that the direction of A is the same as that of the current density. The minus sign in the PDE is also in the voltage equation but does not appear in the solution.