

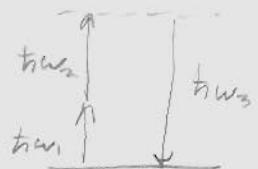
Conservation Laws

photon energy:

$$\left(-\nabla^2 + \frac{n_3^2}{c^2} \frac{\partial^2}{\partial t^2}\right) E_3 e^{-i\omega_3 t} = 4\pi \chi^{(1)} E_1 e^{-i\omega_1 t} E_2 e^{-i\omega_2 t}$$

to cancel time-dep. exponentials,

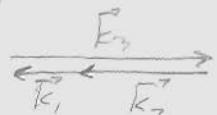
$$\hbar\omega_3 = \hbar\omega_1 + \hbar\omega_2$$



photon momentum

same argument if k 's are all parallel

$$\hbar k_3 = \hbar k_1 + \hbar k_2$$



$$\text{note that } \hbar k_3 = \hbar n(\omega_3) \frac{w}{c}$$

$$\text{if } \underline{\omega_1 = \omega_2},$$

$$\hbar k_3 = 2\hbar k_1$$

$$\text{or } \frac{n(\omega_3) w_3}{c} = 2n(\omega_1) \frac{w_1}{c} \rightarrow \frac{1}{v_{ph(\omega_3)}} = \frac{1}{v_{ph(\omega_1)}}$$

so phase rel. are equal if $\Delta k = 0$

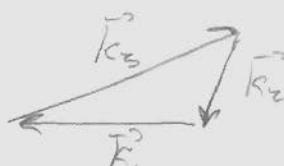
but for SFG this isn't right

$$\frac{w_3}{v_{ph_3}} = \frac{w_1}{v_{ph_1}} + \frac{w_2}{v_{ph_2}}$$

so just remember $\Delta k = \sum k_i$

noncollinear case:

$$\vec{k}_3 = \vec{k}_1 + \vec{k}_2$$



~~beam power: Manning - Rowe~~

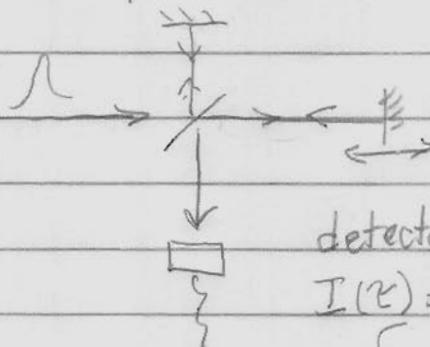
$$\frac{d}{dz} (I) = \frac{d}{dz} (I_2) = - \frac{d}{dz} (I_3)$$

Pulse characterization with $\chi^{(2)}$ materials

Autocorrelation

$$g_{AC}(\tau) = \int_{-\infty}^{\infty} f(t) f^*(t-\tau) dt$$

τ = delay b/w pulses



$$\begin{aligned} I(\tau) &= \langle |E_1(t) + E_2(t-\tau)|^2 \rangle dt \\ &= \int dt (|E_1(t)|^2 + |E_2(t-\tau)|^2 \\ &\quad + 2 \operatorname{Re}(E_1(t) E_2^*(t-\tau))) \end{aligned}$$

$$I(\tau) = E_1^2 + E_2^2 + 2 \operatorname{Re}(g_{AC}(\tau))$$

if beam splitter is 50-50 $E_1 = E_2, E_1 = E_2$

Can we learn anything about the pulse duration or shape from a field autocorrelation?

No: we can get a measure of the power spectrum

$$\langle \{ g_{AC}(\tau) \} \rangle = |F(\omega)|^2 \quad (\text{requires } f(t) \text{ real})$$

this is the basis of FTIR spectrometry.

Alt picture: when time-averaging, interference occurs in

w space

$$I(\omega, \tau) = |E(\omega) + E(\omega)e^{i\omega\tau}|^2$$

$$= |E(\omega)|^2 |1 + e^{i\omega\tau}|^2 \quad \text{indep of phase } \phi(\omega)$$

Spatial effects

- crossed beams
- walkoff
- focusing
- waveguides.

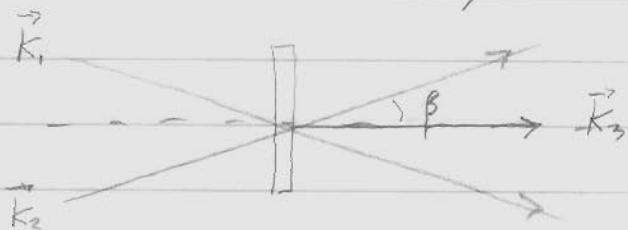
1) crossed beams : plane waves

$$\vec{E}_1 = A_1 e^{i\vec{k}_1 \cdot \vec{r}}, \text{ etc.}$$

$$A_3' \propto A_1 A_2 e^{i\vec{\Delta k} \cdot \vec{r}}$$

$$\vec{\Delta k} = \vec{k}_1 + \vec{k}_2 - \vec{k}_3$$

non-collinear mixing : $\omega_1 = \omega_2$



$$P_{NL} \propto \chi^{(2)} E^2 \propto A_1^2 e^{2i\vec{k}_1 \cdot \vec{r}} + A_2^2 e^{2i\vec{k}_2 \cdot \vec{r}} + A_1 A_2 e^{i(\vec{k}_1 + \vec{k}_2) \cdot \vec{r}} + C.C. + A_1 A_2^* e^{i(\vec{k}_1 - \vec{k}_2) \cdot \vec{r}} + C.C. \quad (\text{D.C.})$$

write eqn for each output wave, direction

SVM mixing:

$$\frac{\omega_1 n_1 \cos \beta}{c} \hat{z} - \frac{\omega_1 n_1 \sin \beta}{c} \hat{x}$$

$$+ \frac{\omega_1 n_1 \cos \beta}{c} \hat{z} + \frac{\omega_1 n_1 \sin \beta}{c} \hat{x}$$

$$\frac{2\omega_1 n_1 \cos \beta}{c} \hat{z} \rightarrow \text{wave in } \hat{z} \text{ direction}$$

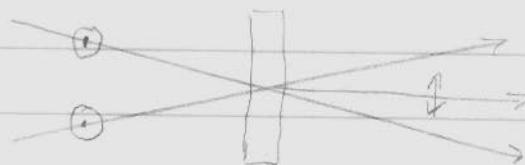
$$\Delta k = \frac{2\omega_1 n_1 \cos \beta}{c} - \frac{\omega_2 n_2}{c} = 0 \rightarrow n_1 \cos \beta = n_2$$

crossing angle lowers effective index

- changes phase matching angle.

two options for type I

1)



$$\vec{E}_1, \vec{E}_2 \parallel \hat{y}$$

and along n_o

normal tuning angle is around β

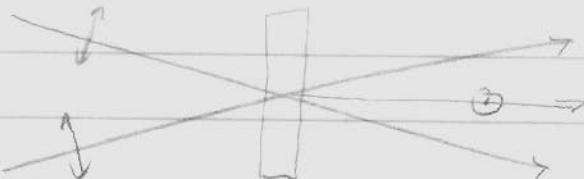
$$n_i \text{ is still } = n_o(w_i)$$

optimum crystal angle is set for

$$n_o(w_i) \cos \beta = n_e(w_i, \theta)$$

In doubling directions, SH is not phase-matched.

2)

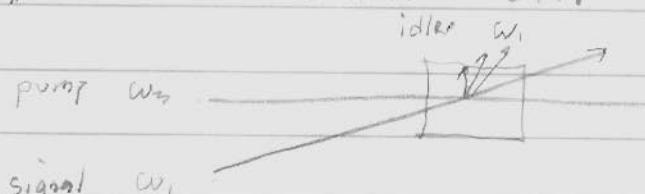


here, the input polarization has a projection on
both n_o and $n_e(\theta)$ axes

- only n_o part will be phase matched

use version 1.

NOPA: non-collinear OPA



idler can come out

over a range of directions

$\rightarrow \Delta k = 0$ over a wide spectral range.

shortest pulses.

Walkoff

consider plane wave normally incident on surface

- input \vec{E} along \vec{S} outside crystal

- input is at angle to crystal axes:



boundary conditions: D^\perp and E^H are continuous.

know that $\vec{E} \nparallel \vec{D}$: one of them bends.

\vec{D}_{in} must be \parallel to \vec{D}_{out}

but $\vec{E}_{out} = \vec{E}_{in} \hat{x}' + E_{out} \hat{z}'$ is ok.

\therefore inside crystal



beam can refract even at normal inc.

but opposite polarization does not refract if it's along
crystal y axis

\rightarrow double image for unpolarized light

two rays walk off from each other: angle β

$$\cos \beta = \frac{\vec{E} \cdot \vec{D}}{E_0 D_0} = \frac{D_x^2 / \epsilon_x + D_y^2 / \epsilon_y + D_z^2 / \epsilon_z}{E_0 D_0}$$

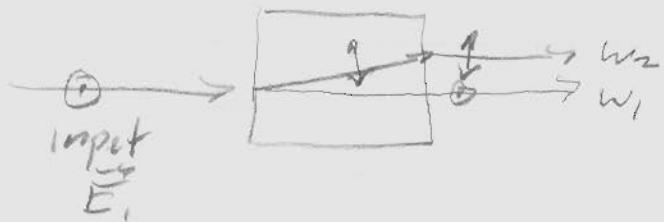
$$\vec{D}_{in} = D_0 \hat{x}' = D_0 \cos \alpha \hat{x} - D_0 \sin \alpha \hat{z}$$

$$\cos \beta = \frac{D_0^2 (\cos^2 \alpha / \epsilon_x + \sin^2 \alpha / \epsilon_z)}{D_0^2 \left(\frac{\cos^2 \alpha}{\epsilon_x} + \frac{\sin^2 \alpha}{\epsilon_z} \right)^{1/2} \left(\cos^2 \alpha + \sin^2 \alpha \right)^{1/2}}$$

calcite: $n_x = 1.658 \quad n_z = 1.486 \quad \alpha = 35^\circ \rightarrow \beta = 6.1^\circ$

Beam walk off.

harmonic propagates w/ \vec{k}^2 \parallel crystal axis,
→ redirection of power flow



if using small beams (e.g. in focus)

→ power walks off, less efficient

Beam power conservation:

$$\frac{d}{dz} \left(\frac{I_1}{w_1} \right) = \frac{d}{dz} \left(\frac{I_2}{w_2} \right) = - \frac{d}{dz} \left(\frac{I_3}{w_3} \right)$$

the Munley-Rome relations come from photon conservation

$$I_j = \frac{1}{2} V_{ph}(w_j) U_j \quad \text{phase vel. energy density}$$

$$= \frac{1}{2} \epsilon_0 \frac{c}{n_j} n_j^2 E_j^2 = \frac{1}{2} \epsilon_0 c n_j E_j^2$$

in terms of photons:

$$I_j = \hbar w_j F_j \quad F_j = \text{photon flux. } \#/\text{area-time}$$

1) use M-R relations to reduce number of equations.

2) power conservation is important for understanding conversion limits in parametric processes.