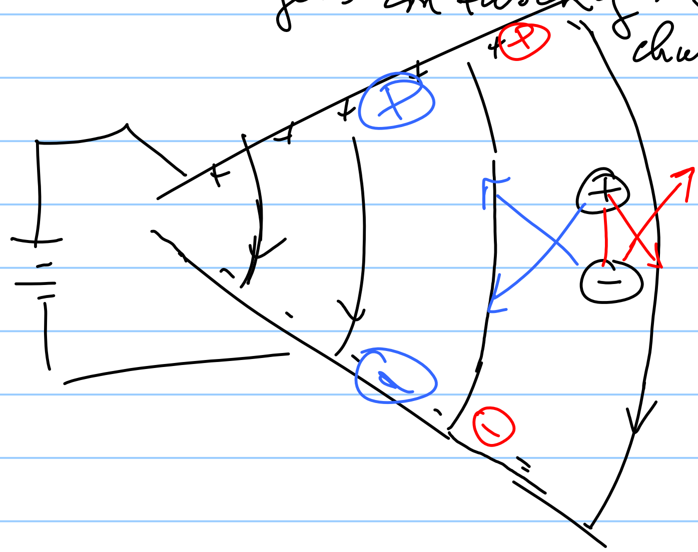
Note Title 2/25/2009

Looking at forces from individual charges on plates gets confusing. Just use $\vec{F} = q\vec{E}$ on each charge of dipole



$\vec{F} = q\vec{E}$

$\vec{F}_{net} = \rightarrow$

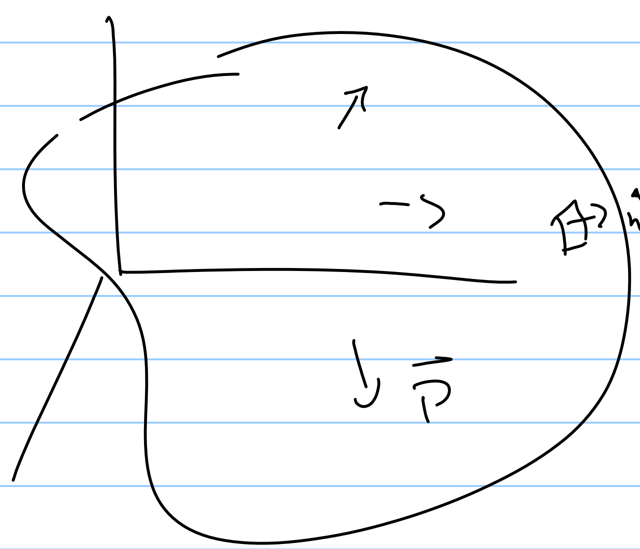
$$F = (\vec{p} \cdot \vec{\nabla})\vec{E}$$

$$\vec{p} = p_0 \hat{z}$$

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\vec{p} \cdot \vec{\nabla} = p_0 \frac{\partial}{\partial z}$$

$$(\vec{p} \cdot \vec{\nabla})\vec{E} = p_0 \frac{\partial}{\partial z}\left(E_x(x,y,z)\hat{x} + E_y(x,y,z)\hat{y} + E_z(x,y,z)\hat{z}\right)$$



$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau}{r}$$

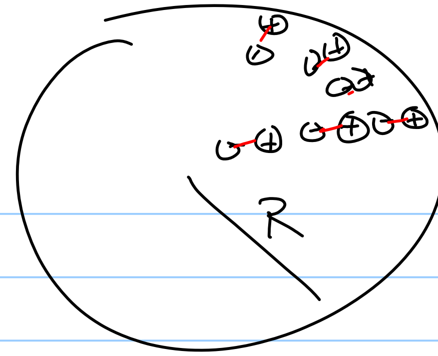
$$= \frac{1}{4\pi\epsilon_0}\left[\int \frac{\sigma_b\, da}{r} + \int \frac{\rho_b\, d\tau}{r}\right]$$

$\sigma_b = \vec{P} \cdot \hat{n}$ ← know

$\rho_b = -\vec{\nabla} \cdot \vec{P}$

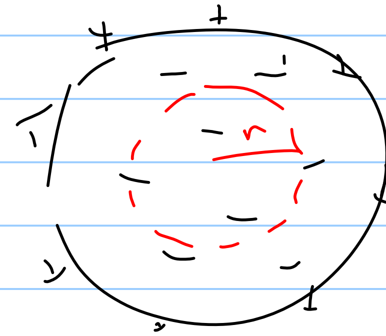
$$\vec{\nabla} \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$

Given $\vec{P} = k r \hat{r}$



$$\sigma_b = k r \hat{r} \cdot \hat{r} \Big|_{r=R} = \underline{kR}$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P} = -\frac{1}{r^2} \frac{\partial (r^2 k r)}{\partial r} + \phi + \phi = -3k$$



$$\oint \vec{E} \cdot da = \frac{Q_{encl}}{\epsilon_0}$$

$$E \, 4\pi r^2 = \frac{\int \overset{-3k}{\rho} \, d\tau}{\epsilon_0}$$

$$E \, 4\pi r^2 = -3k \frac{\frac{4}{3}\pi r^3}{\epsilon_0} \qquad r<R \quad E = \frac{-k r \hat{r}}{\epsilon_0}$$

$r>R$
$$E \, 4\pi r^2 = \frac{Q_{tot}}{\epsilon_0} = \frac{(-3k)\frac{4}{3}\pi R^3}{\epsilon_0} + \frac{4\pi R^2 kR}{\epsilon_0}$$

Real system: What is $\vec{P}$

induced dipole in LINEAR SYSTEM

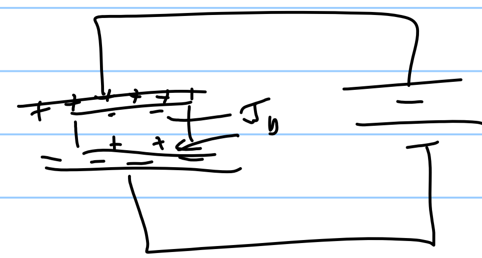
$$\vec{P} \propto \vec{E} \qquad \boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}}$$

Gauss's law $\epsilon_0 \vec{\nabla}\cdot\vec{E} = \rho = \rho_f + \rho_b$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

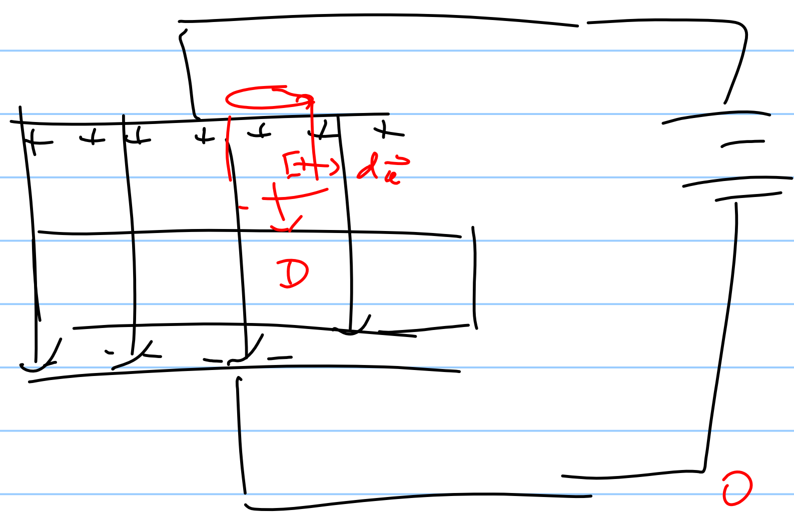
$$\vec{\nabla}\cdot\underbrace{\left(\epsilon_0\vec{E} + \vec{P}\right)}_{\vec{D}} = \rho_f$$

ρ_f ← we put on cap plate



$$\vec{\nabla}\cdot\vec{D} = \rho_f \qquad \oint \vec{D}\cdot d\vec{a} = Q_{f,\,enclose}$$

Ex:



$$\vec{D} = \underline{\epsilon_0}\vec{E} + \vec{P} \qquad \oint \vec{D}\cdot d\vec{a} = \cancel{\int_{\text{upper cap}}}^{\,0} + \cancel{\int_{\text{body}}}^{\,\vec{D}\perp d\vec{a}} + \int_{\text{bottom cap}}$$

$$\int_{\text{bottom cap}} |\vec{D}||d\vec{a}|\underbrace{\cos\phi}_{1} = D\cancel{A} = \sigma_f \cancel{A} \qquad D = \sigma_f$$

Same in glass & vac

IN AIR $\quad D = \sigma_f = \epsilon_0 E + P = \epsilon_0 E = \sigma_f$

zero in vac

IN GLASS

$$D = \sigma_f = \epsilon_0 E + P = \epsilon_0 E + \epsilon_0 \chi_e E$$

$$P = \epsilon_0 \chi_e E$$

$$\sigma_f = (\epsilon_0 + \epsilon_0 \chi_e) E \qquad E = \frac{\sigma_f}{\epsilon_0 (1 + \chi_e)}$$