Note Title 2/12/2010

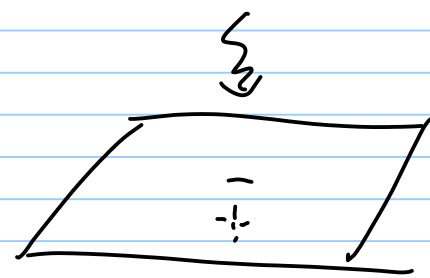
Applications

- Sun's radiation, cosmic rays, .. generate charges in atmosphere

  Thunderstorms sep. these charges ⇒ large E ⊥ surface

  E is largest around the world during afternoon in Brazil

- laser printer



toner particles are charged & deposited on this surface where they stick to surface charge ⇒ image

Look at print with microscope: ink/toner particles

Exam Wed. Feb 24 on sep. variables

next hmwk due Feb 24

practice exam/problems Feb 22

Review of last lecture

$\nabla^2 V = 0$ Assume $V(x,y,z) = X(x)\,Y(y)\,Z(z)$

$\Rightarrow$ 3 ODE's $\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \quad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2 \quad \frac{1}{Z}\frac{d^2Z}{dz^2} = C_3$

$\boxed{C_1 + C_2 + C_3 = 0}$ for Laplace's eqn to be satisfied

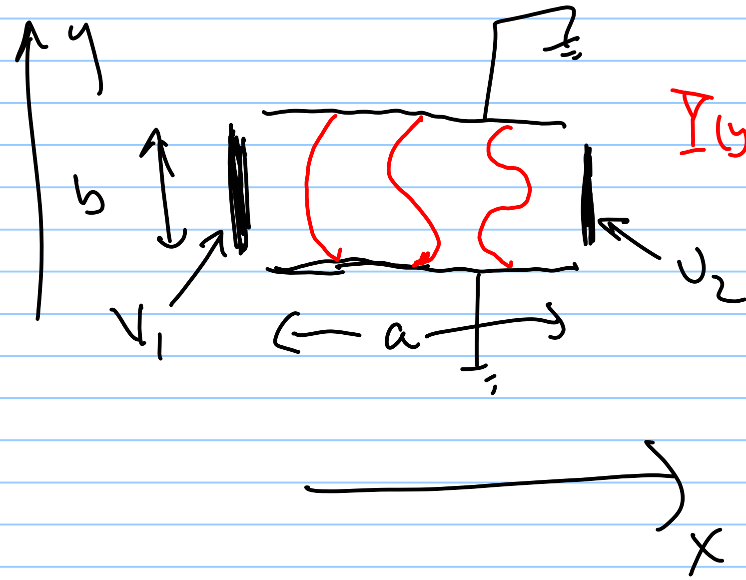
Solns:

$A \sin ky + B \cos ky \qquad c < 0$

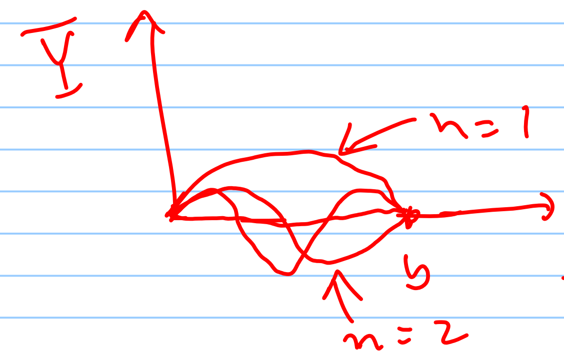
$A' e^{\sqrt{c}\,y} + B' e^{-\sqrt{c}\,y} \qquad c > 0$

$A'' + B'' y \qquad c = 0$

Ex:

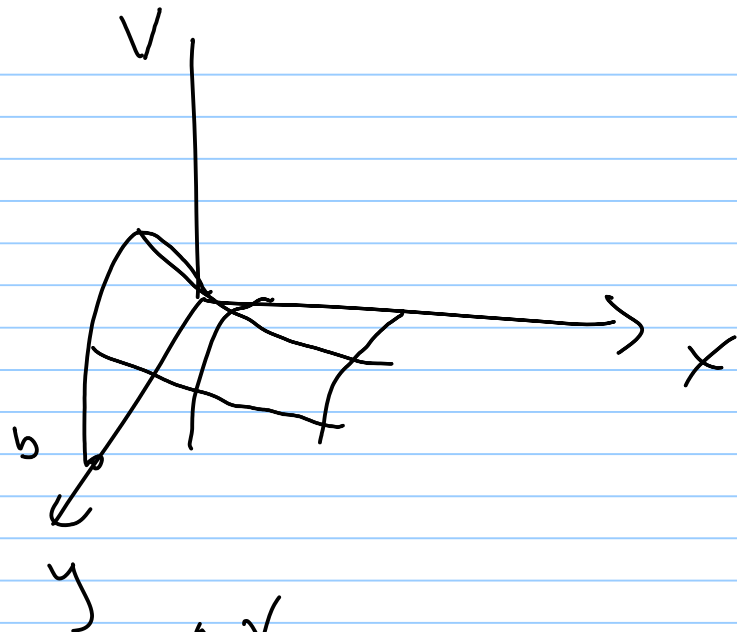


$Y(y) = A \sin ky \qquad k = \frac{n\pi}{b}$



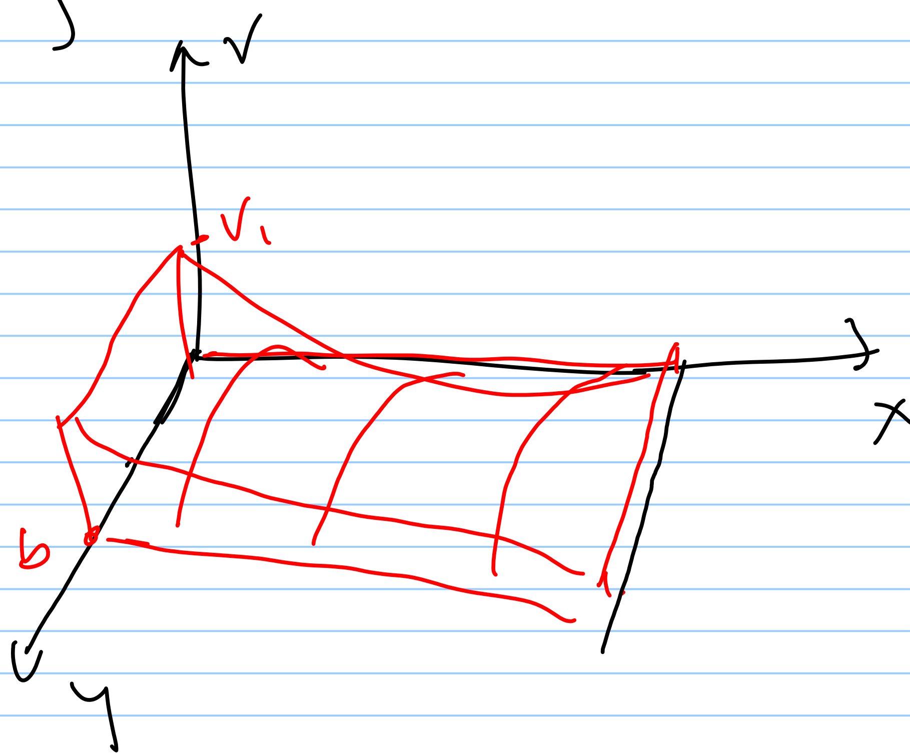
$$V(x,y,z) = X(x)\,Y(y)\,Z(z) = \left(G e^{kx} + H e^{-kx}\right) \sin ky$$

$$k = \frac{n\pi}{b}$$



$n = 1 \quad G = 0$

$$V(x, y, z) = H e^{-\frac{\pi}{b} x} \sin\left(\frac{\pi}{b} y\right)$$



Superposition principle $\quad V = \sum' A_i V_i$

sep. variables solns

$$V(x,y,z) = \sum_{n=1}^{\infty}{}' \left(A_n e^{-\frac{n\pi x}{b}} + B_n e^{\frac{n\pi x}{b}} \right) \sin\left(\frac{n\pi y}{b}\right)$$

Boundary condition is

$$V(x=0) = V_1 = \sum{}' (A_n + B_n) \sin\frac{n\pi y}{b}$$

$$\delta_{nm} = \begin{matrix} 0 & n \neq m \\ 1 & n = m \end{matrix}$$

Fourier's trick $\quad \int_0^b \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{m\pi y}{b}\right) dy = \frac{b}{2}\delta_{nm}$

Mathematica Simplify[Integrate[sin[ ] sin[ ], {y,0,b}], Assumptions → $m \in$ Integers & $n \in$ Integers

multiply both sides of and integrate to apply Fourier's trick

$$\int_0^b V_1 \sin\left(\frac{m\pi y}{b}\right) dy = \sum_n{}' (A_n + B_n) \underbrace{\int_0^b \sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{m\pi y}{b}\right) dy}_{\frac{b}{2}\delta_{mn}}$$

$$\boxed{-\frac{bV_1}{m\pi}\left[\cos(m\pi) - 1\right] = (A_m + B_m)\frac{b}{2}}$$

1 eqn. in 2 unknowns A_m & B_m

$$\left.\begin{array}{ll} m \text{ odd} & \dfrac{2bV_1}{m\pi} \\ m \text{ even} & 0 \end{array}\right\} = (A_m + B_m)\frac{b}{2}$$

Next bndm condition at $x = a$

$$V(x=a) = V_2 = \sum_n \left(A_n e^{-\frac{n\pi a}{b}} + B_n e^{\frac{n\pi a}{b}} \right) \sin\left(\frac{n\pi y}{b}\right)$$

multiply both sides by $\sin\left(\frac{m\pi y}{b}\right)$ & integrate $0 \to b$

$$A_n e^{-n\pi a/b} + B_n e^{n\pi a/b} = \begin{cases} \frac{4V_2}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

2 eqns in A_n & B_n

$$A_n = \frac{4}{n\pi} \left(\frac{V_1 - V_2 e^{-n\pi a/b}}{1 - 2 e^{-2n\pi a/b}} \right)$$

$$B_n = -A_n$$

once we know A_n & B_n we have solved Laplace's eqn

$$V_n(x,y,z) = \sum_{n=\text{odd}} \left(A_n e^{-\frac{n\pi x}{b}} + B_n e^{\frac{n\pi x}{b}} \right) \sin\left(\frac{n\pi y}{b}\right)$$

Need a linear PDE to apply Sep. Var.

$$\underbrace{\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \ldots \right)}_{\text{operator}} \psi(x,t) = 0$$

If ψ_1 is a soln & ψ_2 is a soln

then $\psi_1 + \psi_2 = \psi_{tot}$ is a soln for a linear P.D.E.