Physics 350 – Undergraduate Classical Mechanics
Numerical Homework I, due Friday, September 9, 2011 at 11:00 a.m.

Note: This homework has two pages.

Suppose you are preparing a proposal for NASA to send a new probe to Mars and dig deeper for life using a laser to burn off a layer of the surface ice recently discovered there. Given the limits of your NASA proposal budget, in order to conserve funds for your laser your initial plan is to just drop a probe in a spherical shell from a high place in orbit, with no parachute or active engine but with cushioning on the inside to protect the delicate machinery. As an initial estimate for NASA officials who are considering whether or not to take your proposal seriously, calculate the position, velocity, and acceleration as a function of time during the whole drop for your probe, given that it begins its descent with zero velocity and zero acceleration at a height of one million meters above the mean surface level. You can assume the 10 kg mass in your Mars probe is evenly distributed, and it has a radius of five meters. You should include in your estimate the drag due to Mars' atmosphere and the changing value of the local gravitational acceleration as the probe approaches the surface.

Recall that the gravitational force is

$$F_{\text{grav}} = -\frac{GmM}{r^2}\,. \tag{1}$$

You can take the force the Martian atmosphere exerts on the probe to be

$$F_{\text{atmo}} = \frac{1}{2}C_d\,\rho_{\text{atmo}}(y)Av^2 \tag{2}$$

with y the height above the surface, $C_d = 0.2$ the dimensionless drag coefficient, and A and v the area and velocity of the probe. The local atmospheric density is

$$\rho_{\text{atmo}}(y) = \rho_0\exp(-y/y_0)\,. \tag{3}$$

To solve this problem you will need to look up the mass and mean radius of Mars, as well as determine the mean surface level atmospheric density ρ_0 and and *atmospheric scale height* y_0. You may find
http://en.wikipedia.org/wiki/Atmosphere_of_Mars a useful place to start; take the surface level temperature of Mars to be the average on the polar ice cap, $\simeq -100$ C. For your calculations, follow the explicit outline below.

1. Develop Mathematica code to perform these calculations. Make sure to comment your code well and include it with your solution. An electronic copy of this code must be e-mailed to tguan@mines.edu, as well as a hard copy turned in during class. The e-copy must run on the first pass, without any modifications, and produce all plots and numerical values, or you will not receive full credit. Please note that identical code from two or more persons will result in a score of zero for those parties.

2. Plot $x(t)$, $v(t)$, and $a(t)$ in your Mathematica notebook over the entire time of the probe's drop.

3. Calculate numerical values in correct SI units of the final time, speed, and acceleration at which your Mars probe hits the surface.

4. What is the maximum speed and maximum acceleration during free fall?

5. As a limiting case, show that your expression gets nearly the same value as $y = \frac{1}{2} g_{\text{MarsSurface}} t^2$ if the probe were to start near the ground, at a height of 10 m. Calculate the percent error, given by

$$100 \times \frac{|x_{\text{exact}} - x_{\text{approx}}|}{\frac{1}{2}(x_{\text{exact}} + x_{\text{approx}})} \tag{4}$$

with x_{exact} and x_{approx} the values of final time, velocity, and acceleration given by the exact and approximate expressions, respectively.