Selection rules

The transition rates for electric dipole transitions depend on the dipole moment:

$$\vec{\mu} \propto \langle \psi_b | \vec{r} | \psi_a \rangle$$

For an atomic system, this looks like

$$\langle n'l'm' | \vec{r} | nlm \rangle$$

There are two ways to look at the selection rule:

1) direct integration
2) operators + commutators.

m selection rule

1) inspect integrals w/ actual wavefunctions

m selection rule:

$\psi_{nlm}(\phi) \sim e^{im\phi}$ these are orthogonal

z: $\langle m' | z | m \rangle = r\cos\theta \langle m' | m \rangle$
$= r\cos\theta\, \delta_{m,m'}$

$\therefore \underline{\Delta m = 0}$ for z

x: $\langle m' | x | m \rangle = r\sin\theta \langle m' | \cos\phi | m \rangle$

$$\rightarrow \int_0^{2\pi} e^{-im'\phi} \cdot \tfrac{1}{2}(e^{i\phi} + e^{-i\phi}) e^{im\phi}\, d\phi$$

here you can see that the $\hat{x}$ operator connects states with $\underline{\Delta m = \pm 1}$

y: similar to x

2) operator method:

commutator $[\hat{L}_z, \hat{x}] = i\hbar y$

proof: $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$

$[\hat{x}\hat{p}_y - \hat{y}\hat{p}_x, \hat{x}] = 0 - \hat{y}[\hat{p}_x, \hat{x}] = i\hbar y$

$\therefore \langle n'l'm'|[\hat{L}_z, \hat{x}]|nlm\rangle$

$= \langle n'l'm'|(m'x - xm)\hbar|nlm\rangle$

$(m'-m)\hbar\langle n'l'm'|x|nlm\rangle$

also $= \langle n'l'm'|i\hbar y|nlm\rangle$

similarly $\langle n'l'm'|[L_z, y]|nlm\rangle$

$= (m'-m)\hbar\langle n'l'm'|y|nlm\rangle = -i\hbar\langle n'l'm'|x|nlm\rangle$

using both: $(m'-m)\langle\Psi'|x|\Psi\rangle = i(-i)\dfrac{\langle\Psi'|x|\Psi\rangle}{(m'-m)}$

$\to (m'-m)^2 = 1$

$\therefore \Delta m = \pm 1$

$\therefore$ for random orientation

$\boxed{\Delta m = 0, \pm 1}$

<u>l selection rule</u>

i) integral method:

$\Psi_{nlm}(r,\theta,\phi) \propto Y_l^m(\theta,\phi) \propto P_l^m(\cos\theta)$

$z = r\cos\theta \propto rP_1^0$

$x, y = r\sin\theta \propto rP_1^1$

So to go from the 1s state to any other, the integral looks like

$$\langle n'l'm' | Y_{l=1}^{m} | 100 \rangle$$

which, since $|100\rangle$ is just a constant, is like

$$\langle n'l'm' | n1m \rangle$$

so that $\Delta l = 1$ in this case.

More generally, operators $\hat{x}, \hat{y}, \hat{z}$ can be represented as tensor operators, and the integral

$$\int Y_{l'}^{m'*} \, Y_{1}^{m=0,\pm1} \, Y_{l}^{m} \, d\Omega$$

can be understood in the context of angular momentum addition $j = l, \; j = l-1, \; j = l+1$

2: operator method - see Griffiths.

Summary: $\Delta m = 0, \pm 1$

$\Delta l \; \pm 1$ $\quad \Delta l = 0$ forbidden by parity.

remember: photon is spin 1

interaction carries angular momentum.