Lecture 34 April 17

Note Title 4/17/2006

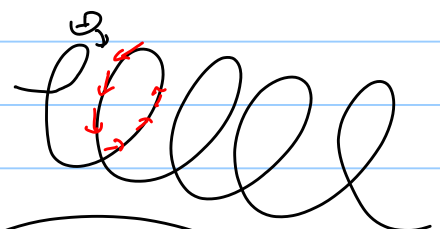
Energy in electric field $\int \frac{1}{2}\epsilon_0 \vec{E}^2 \, d\tau$

" " mag " ? Joules/m^3

$W = q\Delta V = q L \frac{dI}{dt}$

Power $= I\Delta V = I L \frac{dI}{\cancel{dt}} = \frac{dW}{\cancel{dt}}$

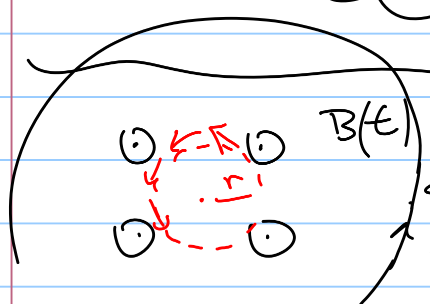
$$\boxed{W = \frac{1}{2} L I^2}$$



$\mathcal{E}mf = -\frac{d\Phi_B}{dt}$

"

$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

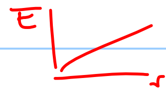


$\oint \vec{E}\cdot d\vec{l} = E\oint |d\vec{l}| = \frac{d}{dt}\int \vec{B}\cdot d\vec{a}$

" $\quad\quad\quad 2\pi r$

$r\,d\theta\,\hat{\theta}$

$= \frac{dB}{dt}\pi r^2$



$E = \frac{dB}{dt}\frac{r}{2}$

$$W = \frac{1}{2} L I^2 \qquad I \to \vec{B} \;\&\; \vec{A}$$

$$\Phi = LI = \int \vec{B}\cdot d\vec{a}$$

same

$$W = \frac{1}{2} \frac{\oint \vec{A}\cdot d\vec{l}}{\cancel{I}} \cancel{I}^{2} \qquad LI = \int \vec{\nabla}\times\vec{A}\cdot d\vec{a}$$

$$LI = \oint \vec{A}\cdot d\vec{l}$$

$$W = \frac{1}{2}\int \vec{A}\cdot\vec{J}\, d\tau$$

all space (only $\vec{J}$ in a small region of space)

Invoke magnetostatics $\vec{\nabla}\times\vec{B} = \mu\vec{J}$ change J to B

$$W = \frac{1}{2\mu_0}\left[\int B^2 d\tau - \oint_S (\vec{A}\times\vec{B})\cdot d\vec{a}\,\right]$$

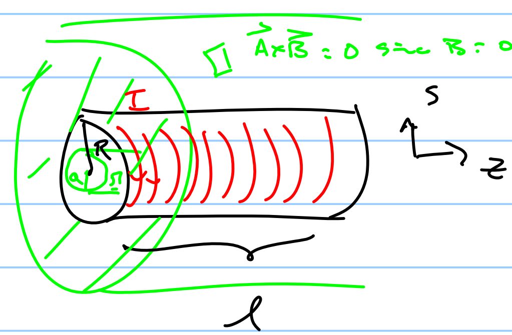
surface enclosing τ

Ex: Solenoid

$$\vec{B} = \mu_0 n I \hat{z}$$

$$\vec{A} = \frac{\mu_0 n I}{2} s\,\hat{\phi} \qquad s<R$$

$$\vec{A} = \frac{\mu_0 n I}{2} \frac{R^2}{s}\hat{\phi} \qquad s>R$$



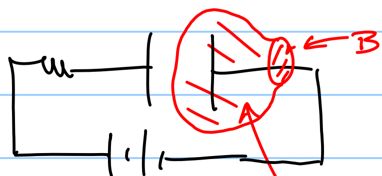
Maxwell's Eqns

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\int\vec{\nabla}\times\vec{B}\cdot d\vec{a} = \int\mu_0\vec{J}\cdot d\vec{a}$$

$$\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$



no $I_{enclosed}$

for all vector functions

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}} \qquad \vec{\nabla}\cdot(\vec{\nabla}\times\vec{C}) \equiv 0 \qquad \vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B}) = 0$$

magnetostatics $= 0$

oh

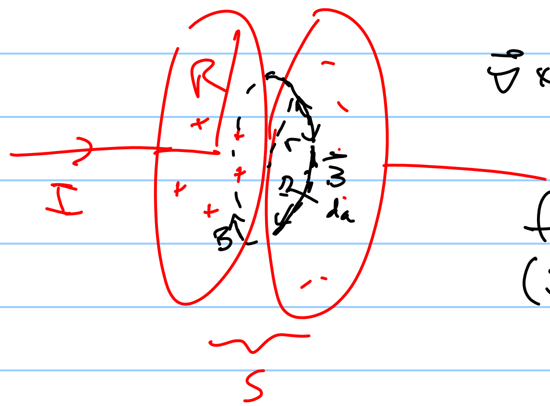
$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\vec{\nabla}\cdot\vec{J} \neq 0$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$ addition Maxwell

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \vec{\nabla}\cdot\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$= \mu_0 \vec{\nabla}\cdot\vec{J} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\underbrace{\vec{\nabla}\cdot\vec{E}}_{\frac{\rho}{\epsilon_0}} = \mu_0\Big(\underbrace{\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t}}_{0}\Big) = 0$$

by cons of charge



$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

find E in between plates

$(J = 0)$

$s \ll R$

integral form $\quad \vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

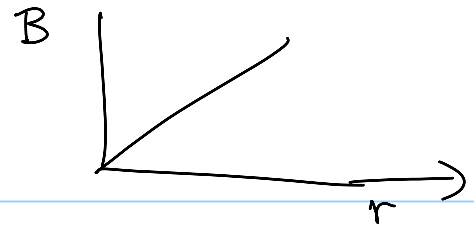
$$\int \vec{\nabla}\times\vec{B}\cdot da = \mu_0\epsilon_0\int\frac{\partial E}{\partial t}\cdot da$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} \qquad \oint \vec{B}\cdot d\vec{l} = \mu_0\epsilon_0\frac{\partial}{\partial t}\int \vec{E}\cdot d\vec{a}$$

$$\frac{\partial E}{\partial t} = \underbrace{\frac{1}{\epsilon_0}\frac{I}{A}}\, = \frac{J}{\epsilon_0} \qquad B\,2\pi r = \mu_0\epsilon_0\int\vec{J}\cdot da = \mu_0\cancel{\epsilon_0}\frac{1}{\cancel{\epsilon_0}}\frac{I}{\pi R^2}\underbrace{\int da}_{\pi r^2}$$

assumes J is constant

$$B\,2\cancel{\pi}r = \mu_0 I\frac{\cancel{\pi}r^2}{\pi R^2} \qquad B = \frac{\mu_0 I}{\pi\, 2}\frac{r}{R^2}$$


B
r