

### 1. (10 Points) Conceptual Questions

- 1

2. (10 Points) Given the following,

$$\frac{2 \sin(n\pi)}{n} = \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \quad 0 = \int_{-\pi}^{\pi} f(x) dx, \quad \frac{2 \cos(n\pi)}{n} = \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$
$$10 = \int_{-\pi}^{\pi} g(x) dx, \quad \frac{2(e^{in\pi} - e^{-in\pi})}{n^2} = \int_{-\pi}^{\pi} g(x) e^{-inx} dx$$

where  $n$  is an integer:

(a) Find the real Fourier series of  $f$ . Is  $f$  even or odd?

(b) Find the complex Fourier series of  $g$ . Is  $g$  even or odd?

3. (10 Points) Given that the coefficients of a complex Fourier series are  $c_0 = 0$  and  $c_n = \frac{2(-1)^n}{n^2}$ , find the real Fourier series.

4. (10 Points) Given,

$$f(x) = A, \quad x \in (0, a), \quad (1)$$

calculate the Fourier cosine and Fourier sine half-range expansions of  $f$ .

5. (10 Points) Given,

$$f(x) = \begin{cases} -x, & -1 < x < 0 \\ x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}, \quad (2)$$

calculate  $\hat{f}(\omega)$ .

EXTRA CREDIT MATERIAL  
FINISH EXAM - PROCEED WITH CAUTION

Public choice theory is a branch of economic theory beginning in 1948 by the work of Duncan Black. These concepts can be applied to study the problems of politics and give rise to the concept of median voter theory. The median voter theorem (MVT) states:

- In a majority election, if voter policy preferences can be represented as a points along a single dimension, if all voters vote deterministically for the politician that commits to a policy position closest to their own preference, and if there are only two politicians, then if the politicians want to maximize their number of votes they should both commit to the policy position preferred by the median voter.

Which is to say that in equilibrium:

- Voters cast for either candidate with equal probability
- In expectation each politician receives half of the votes.
- If either candidate deviates to commit to a different policy position, the deviating candidate receives less than half the vote.

We can take this to mean that in equilibrium contests tend to ties. There are arguments that the 2000 and 2004 elections are indications that this is currently going on. Some have even implied this mechanism is necessary for functioning democracy.

**Question 1:** Do you feel that this election supports or refutes the claim that American politics obeys the median voter theory?

**Question 2:** Do you feel that the winner of yesterdays contest changed the voter distribution or appealed to existing median voters?

**Question 3:** Do you feel that the median voter is representative of your policy positions?

**Question 4:** In a two party system each candidate typically appeals to the median voter of his/her voter-distribution/constituency, but in the case of a national election they must appeal to a larger different distribution. Using **convolution** integrals it is possible to ‘mix’ these voter distributions. How would one ‘mix’ in a three-party system?