Electric dipole radiation
- small source $d \ll \lambda$ $\beta \ll 1$ $K \approx 1$

Add up fields from each charge

$$(\vec{E}_a)_\alpha = \frac{q_\alpha}{c^2}\left[\frac{\hat{R}_\alpha \times (\hat{R}_\alpha \times \vec{a}_\alpha)}{R_\alpha}\right]$$

with one source charge → Larmor formula.

add up sources.

if we ignore retardation w/in source (except for $\vec{a}_\alpha$)
$R \sim r$ since $r \gg d$

$$\rightarrow \vec{E}_{rad} = \sum_\alpha (\vec{E}_a)_\alpha = \frac{1}{c^2 r}\hat{r} \times \left(\hat{r} \times \left[\sum_\alpha q_\alpha \vec{a}_\alpha\right]\right)$$

express in terms of dipole moment:

$$\vec{p} = \sum_\alpha q_\alpha \vec{r}_\alpha{}' \quad \text{(prime w/in source)}$$

$$\ddot{\vec{p}} = \sum_\alpha q_\alpha \vec{a}_\alpha$$

$$\rightarrow \vec{E}_{rad} = \frac{1}{c^2 r}\hat{r} \times (\hat{r} \times [\ddot{\vec{p}}])$$

we can use same Larmor formula:

$$\frac{dP}{d\Omega} = \frac{e^2[a^2]}{4\pi c^3}\sin^2\theta \rightarrow \frac{[\ddot{p}^2]}{4\pi c^3}\sin^2\theta$$

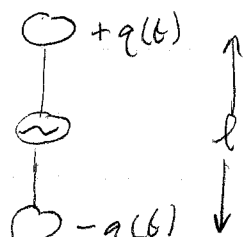
Time dependence $p(t) \sim p_0 e^{-i\omega t}$ or $p_0 \cos\omega t$

$$\rightarrow \langle \ddot{p} \rangle \quad p_0^2 \omega^4 \langle \cos^2 \omega t \rangle = \frac{1}{2} p_0^2 \omega^4$$

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{p_0^2 \omega^4}{8\pi c^3}\sin^2\theta \qquad \langle P \rangle = \frac{p_0^2 \omega^4}{3c^3}\,!$$

Complete fields from an osc. dipole - summary

model:



current $I = dq/dt$

dipole moment $p = q(t)\cdot \ell$

$\rightarrow I = \dot{p}/\ell$

here, start w/ known charges + currents.

$\vec{B}$ is fcn of $\vec{J}$:

$$\vec{J}\,d^3r = \vec{J}\,d^2r\cdot d\ell = I\,d\ell = \dot{p}\,\frac{\vec{\ell}}{\ell} = \dot{\vec{p}}$$

$$\therefore\ \vec{B}(\vec{r},t) = \int_V \left(\frac{[\vec{J}]\times\hat{r}}{cR^2} + \frac{[\partial\vec{J}/\partial t]\times\hat{r}}{c^2R}\right)d^3r$$

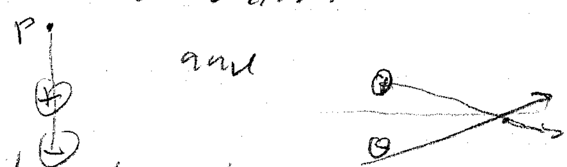
$$= \frac{[\dot{\vec{p}}]\times\hat{r}}{cr^2} + \frac{[\ddot{\vec{p}}]\times\hat{r}}{c^2r} = \left(\frac{[\dot{p}]}{cr^2} + \frac{[\ddot{p}]}{c^2r}\right)\sin\theta\,\hat{\phi}$$

for the E-field,

$$\vec{E}(\vec{r},t) = \int\left(\underbrace{\frac{[\rho]\,\hat{r}}{R^2} + \frac{[\dot{\rho}]\,\hat{r}}{cR}}_{\sum_\alpha\left(\frac{[q_\alpha]\,\hat{r}}{r^2} + \frac{[\dot{q}_\alpha]\,\hat{r}}{cr}\right)} - \underbrace{\frac{[\dot{\vec{J}}]}{c^2R}}_{-\frac{[\ddot{\vec{p}}]}{c^2r}}\right)d^3r$$

here we must account for retardation

limiting cases and



then extend result to arb angle.

approx $\ell \ll r$

$$\rightarrow \vec{E}(\vec{r},t) = \left(\frac{2[p]}{r^3} + \frac{2[\dot{p}]}{cr^2}\right)\cos\theta\,\hat{r} + \left(\frac{[p]}{r^3} + \frac{[\dot{p}]}{cr^2} + \frac{[\ddot{p}]}{c^2r}\right)\sin\theta\,\hat{\theta}$$

terms $1/r^3$: only $\vec{E}$, static dipole

$1/r$: E_θ, B_ϕ, radiation field

$1/r^2$: all compon. intermediate, induction zone

power radiated from dipole was

$$\langle P \rangle = \frac{p_0^2 \omega^4}{3c^3}$$

$$I = \dot{p}/l = -\frac{i\omega p_0}{l} e^{-i\omega t} = I_0 e^{-i\omega t}$$

$$\therefore |p_0| = I_0 l/\omega$$

$$\langle P \rangle = \frac{I_0^2 l^2 \omega^2}{3c^3} = \frac{4\pi^2}{3c} I_0^2 \left(\frac{l}{\lambda}\right)^2$$

for a resistor, dissipated power is (AC)

$$\langle P \rangle = \frac{1}{2} I_0^2 R$$

$\rightarrow$ "radiation resistance"

$$R_{rad} = \frac{8\pi^2}{3c}\left(\frac{l}{\lambda}\right)^2$$

$$= \frac{2\pi}{3}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{l}{\lambda}\right)^2 = 789\left(\frac{l}{\lambda}\right)^2 \text{ ohms.}$$

but $l/\lambda \ll 1$ $\rightarrow$ inefficient radiator.

$\therefore$ use antenna with $l \sim \lambda$