Note Title 2/9/2009

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2} = 0$$

multiply by $r^2\sin^2\theta$

Let $V(r,\theta,\varphi) = R(r)\,\Theta(\theta)\,\Phi(\varphi)$ put into

divide by V

$$\underbrace{\frac{\sin^2\theta}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{\sin\theta}{\Theta}\frac{d}{d\theta}\sin\theta\frac{d\Theta}{d\theta}}_{m^2} + \underbrace{\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}}_{-m^2} = 0$$

$$\frac{d^2\Phi}{d\varphi^2} = -m^2\Phi \qquad \Phi = A\sin m\varphi + B\cos m\varphi$$

$$\Phi = A e^{\pm im\varphi} = A e^{\pm im(\varphi+2\pi)}$$

$\cos m\varphi \pm i\sin m\varphi$

V must be single valued

$$\Phi(\varphi) = \Phi(\varphi+2\pi) \qquad e^{\pm im2\pi} = 1 \qquad m \text{ is an integer}$$

$$\Phi(\varphi) = \text{const}$$

Azimuthal symmetry $\frac{d^2\Phi}{d\varphi^2} = 0 \Rightarrow m^2 = 0$

$$\nabla^2 V = \frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = 0$$

Assume $V = R(r)\,\Theta(\theta)$ & divide by V

$$\underbrace{\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)}_{k} + \underbrace{\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)}_{-k} = 0$$

$$k - k = 0$$

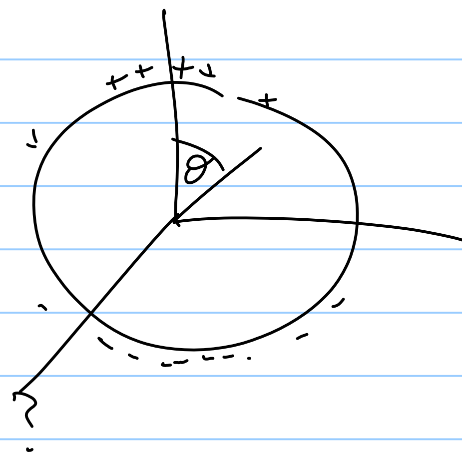
$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = k \qquad \frac{1}{\Theta \sin\theta}\frac{d}{d\theta}\left(\sin\theta\, \frac{d\Theta}{d\theta}\right) = -k$$

$\ell(\ell+1)$ $\quad$ ℓ integer

2nd order ODE's $\Rightarrow$ 2 solns

$$R(r) = A r^{\ell} + B r^{-(\ell+1)} \qquad \Theta(\theta) = P_\ell(\cos\theta)$$

let $x = \cos\theta$ $\qquad P_\ell(x)$



$P_0 = 1 \qquad P_1 = x = \cos\theta$

What does $\sigma = \sigma_0 P_1(\cos\theta)$ look like?

Orthogonality:

$x = \cos\theta$

$$\int_{-1}^{1} P_\ell(x)\, P_m(x)\, dx = \int_0^{\pi} P_\ell(\cos\theta)\, P_m(\cos\theta)\, \underbrace{\sin\theta\, d\theta}$$

$$= \frac{2}{2m+1}\,\delta_{\ell,m}$$

Sep. variable soln $\qquad \left(A r^{\ell} + B r^{-(\ell+1)}\right) P_\ell(\cos\theta)$

Gen soln:

$$\boxed{V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + B_l r^{-(l+1)}\right) P_l(\cos\theta)}$$

find A_l & B_l then we are done

Bndry condition

$\frac{1}{4\pi\epsilon_0}\frac{q}{r}$

constant

$$-\frac{\hbar^2}{2m}\nabla^2\psi + qV(r)\psi = E\psi$$

$R\Theta$ $R\Theta$

$$\nabla^2\psi = \left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)\right] / R\Theta$$

insert

$$\psi(r,\theta) = R(r)\,\Theta(\theta)$$

& divide by ψ

$$\underbrace{\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + qV(r)}_{C_1} + \underbrace{\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)}_{C_2} = E$$

const

$$C_1 + C_2 = E$$

$V(\theta)$ given by

$\Delta V = IR$