2. We have the solution to the homogeneous system to be given by,

$$\left[\begin{array}{ccc|c} 1 & 3 & -5 & 0 \\ 1 & +4 & -8 & 0 \\ -3 & -7 & 9 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & -6 & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$$ x_3 is free $\Rightarrow$ solutions Exist but are non-unique

The vector parametric for is then givn by

$$\begin{aligned} X_3 &= t \\ x_2 &= 3t \\ x_1 &= -3x_2 + 5t = -9t + 5t = -4t \end{aligned} \Rightarrow \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} t = \vec{x}_h \quad (1)$$

The solution to the nonhomogeneous is then given by,

$$\left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{aligned} x_1 &= -4x_3 - 5 \\ x_2 &= 3 + 3x_3 \\ x_3 &= x_3 \end{aligned} \Rightarrow$$

$$\Rightarrow \vec{x} = \begin{bmatrix} -5 - 4x_3 \\ 3 + 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = x_p + x_h, \quad x_p = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} \quad (2)$$

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Comparing (1) and (2) we see that the solutions to the homogeneous solutions are a line in $\mathbb{R}^3$. The nonhomogeneous solutions are a line in $\mathbb{R}^3$ parallel to (1) but going through $(-5, 3, 0)$.