Note Title 3/19/2008

Generalized uncertainty principle.

Let A + B be any 2 observables. Then

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [A,B] \rangle \right)^2$$

proof: sec. 3.5.1 in Griffiths.

This implies that commuting observables can be simultaneously (and independently) measure A and B.

E.g. $[\hat{x}, \hat{p}] = i\hbar$

$$\Rightarrow \quad \sigma_x^2 \sigma_p^2 \geq \left(\frac{1}{2i} i\hbar \right)^2 = \left(\frac{\hbar}{2} \right)^2$$

or $\sigma_x \sigma_p \geq \hbar/2$

NB if $\hat{Q} = [\hat{A}, \hat{B}]$ for $\hat{A}, \hat{B}$ Hermitian, then $\hat{Q}^{+} = -\hat{Q}$

i.e. commutators of Hermitian operators

are anti-Hermitian. Further

$$\langle Q \rangle = \langle \psi | \hat{Q} \psi \rangle$$

$$= \langle \hat{Q}^{+} \psi | \psi \rangle$$

$$= - \langle Q \psi | \psi \rangle$$

$$= - \left(\langle \psi | Q \psi \rangle \right)^{*} \qquad \langle A | B \rangle = \langle B | A \rangle^{*}$$

$$= - \langle Q \rangle^{*}.$$ so $\langle Q \rangle$ is imaginary.

QM in 3D

$$H = \frac{p^2}{2m} + V(x) \Rightarrow \frac{\vec{p}^{\,2}}{2m} + V(x, y, z)$$

$$p_x \to -i\hbar \frac{\partial}{\partial x} \qquad p_y \to -i\hbar \frac{\partial}{\partial y}$$

$$p_z = -i\hbar \frac{\partial}{\partial z}$$

$$\vec{p} \to -i\hbar \nabla$$

so $$\frac{p^2}{2m} \to -\frac{\hbar^2}{2m} \nabla^2$$

3-24-08

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TDSE $\Rightarrow$ $$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi + V\Psi$$

Laplacian

Normalization $$\int |\Psi(x,y,z)|^2 \, d^3r = 1$$

stationary states

$$\Psi_n(\vec{r},t) = \psi_n(r)\, e^{-iE_n t/\hbar}$$

where $H\psi_n = E\psi_n$

$\Rightarrow$ general solution

$$\Psi(\vec{r},t) = \sum c_n \psi_n(\vec{r})\, e^{-iE_n t/\hbar}$$

Review Sep. of var. for ∇^2 in spherical coord.

4.1.1 in Book.

or 11/17/06 from 311 notes

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Future HW:

Let $r_1 = x \quad r_2 = y \quad r_3 = z$

$p_1 = p_x \quad p_2 = p_y \quad p_3 = p_z$

Then $[r_i, p_j] = -[p_i, r_j] = i\hbar\,\delta_{ij}$

$[r_i, r_j] = [p_i, p_j] = 0$

$\sigma_{r_i}\sigma_{p_i} \geq \hbar/2 \qquad \{\text{e.g. } \sigma_x \sigma_{p_x} \geq \hbar/2\}$

No restrictions on $\sigma_x \sigma_{p_y}$ for instance.

Sep. of variables for

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

first we'll tackle ∇^2 in spherical coordinates so that we can consider spherically symmetric potentials (e.g. electron in coulomb field of proton.. Hydrogen).

Digression 3-body problem