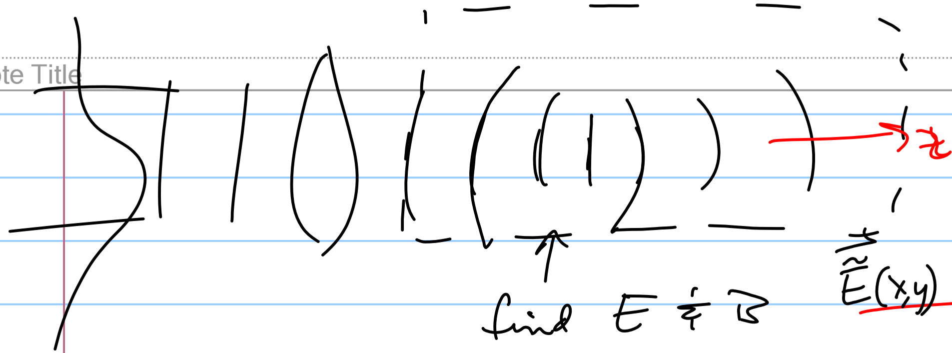


$\tilde{\vec{E}}(x,y)\, e^{i(kz-\omega t)}$

Maxwells Eqns in vac

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t}$$

(1) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\qquad$ $\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\vec{\nabla}\cdot\vec{E} = 0$ $\qquad$ $\vec{\nabla}\cdot\vec{B} = 0$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\,\vec{\nabla}\times\vec{B} = -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}\right) = -\mu_0\epsilon_0\frac{\partial^2 \vec{E}}{\partial t}$$

$$\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{0}) - \nabla^2\vec{E} = -\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \Rightarrow \boxed{\nabla^2\vec{E} = \underbrace{\mu_0\epsilon_0}_{=\frac{1}{c^2}}\frac{\partial^2\vec{E}}{\partial t^2} \qquad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}}$$

put into wave eqn

$$\vec{E} \Rightarrow \tilde{\psi}(x,y,z)\, e^{i(kz-\omega t)} = E(x,y\,z,t)$$

$\uparrow$ scalar

$$\underbrace{e^{i\left(\frac{2\pi}{\lambda} z - \omega t\right)}}_{\text{changes quickly for small changes in } z}$$

ψ slowly changes with z.

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2}(i\omega)^2 E = -\frac{\omega^2}{c^2} E$$

$$k^2 \psi e^{i(kz-\omega t)}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2_{\text{traverse}} E + \frac{\partial^2}{\partial z^2} E + \frac{\omega^2}{c^2} E = 0 \qquad c \to \frac{c}{n}$$

$$E = \psi e^{i(kz-\omega t)} \qquad \frac{\partial E}{\partial z} = \left(ik\psi + \frac{\partial \psi}{\partial z}\right) e^{i(kz-\omega t)}$$

$$\frac{\partial^2 E}{\partial z^2} = \left(-k^2\psi + i2k\frac{\partial \psi}{\partial z} + \frac{\partial^2 \psi}{\partial z^2}\right) e^{i(kz-\omega t)}$$

$$\Rightarrow \boxed{\nabla_t^2 \psi + i\, 2k \frac{\partial \psi}{\partial z} + \frac{\partial^2 \psi}{\partial z^2} = 0}$$

$\times 10^7$

Exact eqn in E becomes PDE in ψ

ψ varies slowly with z so $\frac{\partial \psi}{\partial z} > \frac{\partial^2 \psi}{\partial z^2}$

$k = \frac{2\pi}{\lambda} \approx 10^7$

$$\boxed{\nabla_t^2 \psi + i\, 2k \frac{\partial \psi}{\partial z} = 0}$$ approx

cylindrical

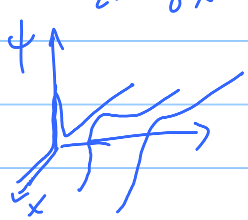
$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \varphi^2}$$

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \psi}{\partial r}\right) + i\, 2k \frac{\partial \psi}{\partial z} = 0$$

guess soln $\psi \propto \exp\left[i \left[P(z) + \frac{k r^2}{2 q(z)}\right]\right]$

$\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} = -i\hbar \frac{\partial \psi(x,t)}{\partial t}$ free particle Sch. eqn

$t \to z$



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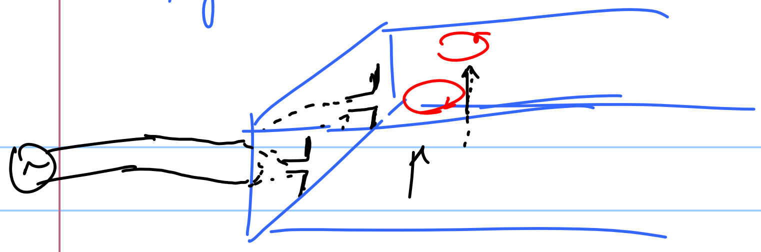
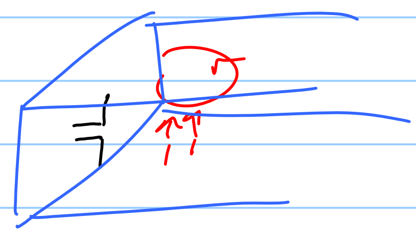
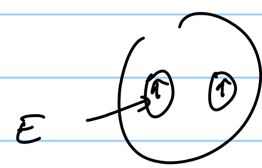
Waveguides:

TEM$_{20}$



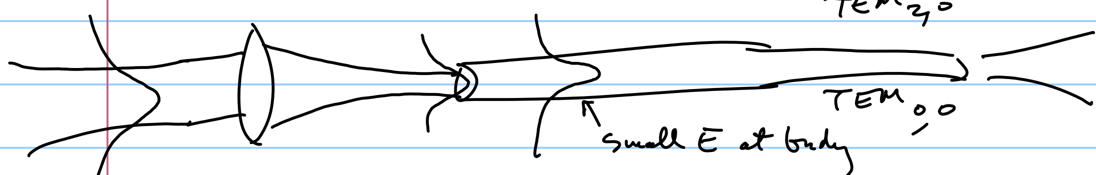
similar





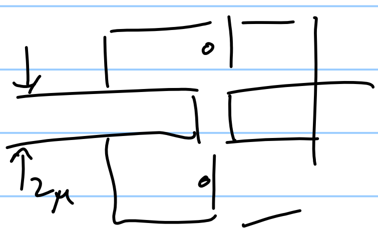
cross section of fiber

TEM$_{20}$

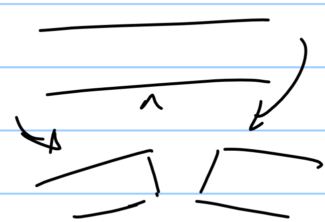


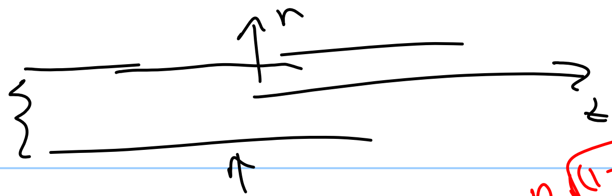
TEM$_{00}$

small E at body



BNC





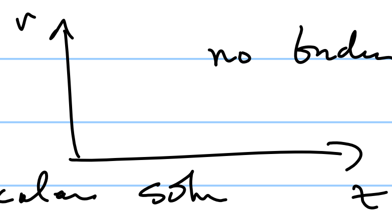
fiber $n(r) = n_0(1 - r^2/\ell^2)$ gradient index

$$\vec{\nabla}^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow \boxed{\nabla^2 \vec{E} + \frac{\omega^2}{c^2} n_0^2 \left(1 - \frac{r^2}{\ell^2}\right)\vec{E} = 0}$$

$c \to \frac{c}{n}$

PDE

characteristic length



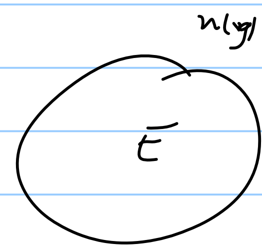
no boundary

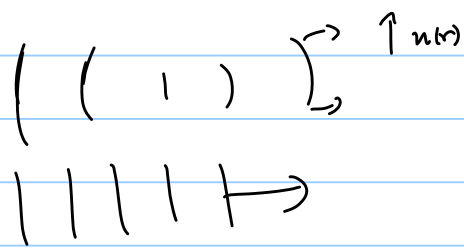
$n^2 = n_0^2\left(1 - \frac{r^2}{\ell^2}\right)$

assume scalar soln

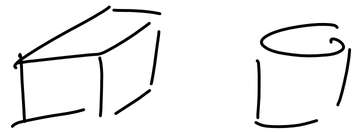
$$E = \psi(x,y)\, e^{i(kz - \omega t)}$$

into





Cavities or resonators



Maxwell's Eqn

assume $\vec{E} = \tilde{\vec{E}}_0(x,y,z)\, e^{-i\omega t}$ $\quad \vec{B} = \tilde{\vec{B}}_0(x,y,z)\, e^{-i\omega t}$

Bndy $\quad E_{\parallel} = 0 \quad B_{\perp} = 0$

$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{\nabla}\times\tilde{\vec{E}}_0 = i\omega\vec{B}$

$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{\nabla}\times\tilde{\vec{B}}_0 = -\frac{i\omega}{c^2}\tilde{\vec{E}}_0$

ME $\longrightarrow$ wave eqn $\quad$ I $\quad \nabla^2 E_x = -\left(\frac{\omega}{c}\right)^2 E_x \leftarrow$ Solve

$$\nabla^2 E_y = -\left(\frac{\omega}{c}\right)^2 E_y$$

$$\nabla^2 E_z = -\left(\frac{\omega}{c}\right)^2 E_z$$

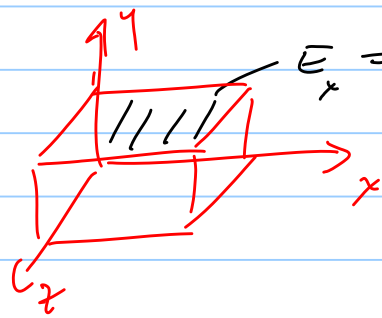
$E_x(x,y,z) = X(x)\, Y(y)\, Z(z)$

Sep const $\quad -k_x^2 - k_y^2 - k_z^2 = -\left(\frac{\omega}{c}\right)^2$

Dispersion relation

$$E_x(x,y,z) = \underbrace{(A\sin k_x x + B\cos k_x x)}_{X(x)}\underbrace{(C\sin k_y y + D\cos k_y y)}_{Y(y)}(\quad)$$

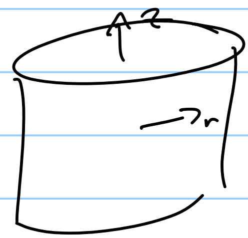
Bndy cond ($E_{\parallel} = 0$) yield some of A, B - - .



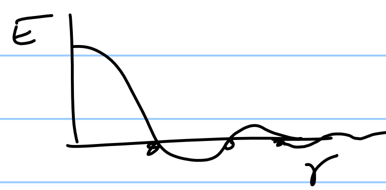
$E_x = 0$ at this bndy

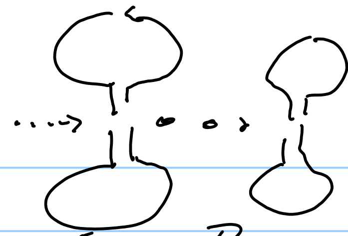
finally look $\vec{\nabla}\cdot\vec{E} = 0$

Can resonator: coords cylindrical



$\Rightarrow$ Bessel functions





ν_0

L

ν:

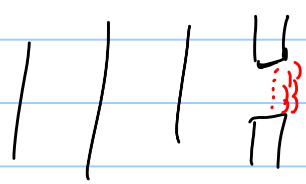
$2L = m\lambda$ $\qquad \lambda\nu = v_p$

$2L = m\, v_p/\nu$ $\qquad \nu = \frac{m\, v_p}{2L}$

Diffraction: wave eqn $\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$

→ assume scalar soln.

→ Mathematical justification for Huygens principle (1704)



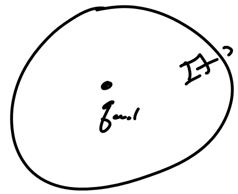
Sum (integrate) fields at aperture to get fields at P

$\times P$?

field at point P in terms of surface integral

~1870

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}$$



- Different from work we did on antennas: - no dipole just pt source
  - vector field before are not treated. We just treat scalar fields