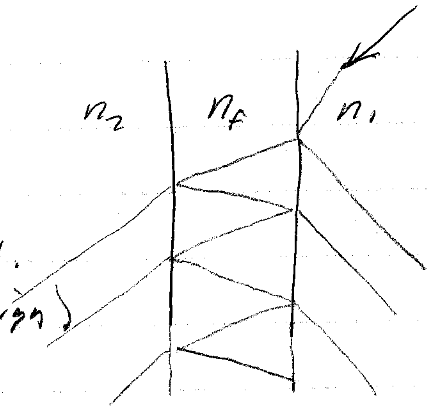
Fabry - Perot

- can solve geometrically, using r, t at each reflection (explicit Fresnel)
- can solve w/ EM + boundary conditions
  - > Fresnel equations implicit



geometric: re: Hecht

r, t at n_1 to n_f interface.

r', t' at n_f to $n_2 = n_1$ interface.

☆ these carry phase and ampl. (incl. sign)

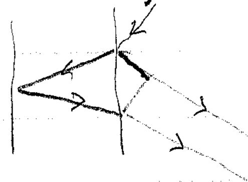
input E_0

reflected $E_0 r$, $E_0 t r' t'$, $E_0 t (r')^3 t'$, ... $\underline{E_0 t (r')^n t'}$
odd n

transmitted $E_0 t t'$, $E_0 t (r')^2 t'$, ..., $\underline{E_0 t (r')^n t'}$
even n

propagation phase $e^{i k_0 \Lambda}$ factor

- optical path for each round trip: $\Lambda = 2 n_f d \cos\theta_t$
  - factor of 2 b/c it is a round trip.
  - $\cos\theta_t$ accounts for angle inside and out side as before:



some Fresnel eqn. relations:

$\underline{\underline{r = -r'}}$ example: $r_\perp = \dfrac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i - n_t \cos\theta_t}$

$n_i \leftrightarrow n_t$ changes sign. → indep. of sign convention

this is that π phase shift

$$\underline{tt' = 1 - r^2}$$

for s tangential E is continuous

$t = 1 + r \qquad (E_t = E_i + E_r)/E_i$

$t' = 1 + r' = 1 - r$

$\therefore\ tt' = (1+r)(1-r) = 1 - r^2$

<u>for p</u>

$$\left.\begin{aligned} 1 + r &= \frac{n_t t}{n_i} \\ 1 + r' &= \frac{n_i t'}{n_t} \end{aligned}\right\} \text{same result.}$$

first look at two special cases

1) $\Lambda = m\lambda_0$ r.t. path = integer λ inside. (one way = $m\lambda/2$)

2) $\Lambda = (m+\frac{1}{2})\lambda_0$ extra $\lambda_0/2$

1) $e^{ik_0\Lambda} = e^{i2\pi m} = 1$

$$\begin{aligned} E_{refl} &= E_0 (r + tr't' + t(r')^3 t' + \dots) \\ &= E_0 r + E_0 r' tt' \underbrace{(1 + (r')^2 + (r')^4 + \dots)}_{1/1-(r')^2} \\ &= E_0 r - \frac{E_0 r (1 - r^2)}{1 - r^2} = 0 \end{aligned}$$

$\overline{1^{st}\ \text{refl}} - \overline{\text{sum(all other)}} = 0$

$\therefore$ full transmission even if r is close to 1!

2) $\Lambda = (m+\frac{1}{2})\lambda_0 \rightarrow e^{2\pi(m+\frac{1}{2})i} = e^{i\pi} = -1$

$$\begin{aligned} E_{refl} &= E_0 (r + tr't' e^{ik_0\Lambda} + t(r')^3 t' e^{2ik_0\Lambda} + \dots) \\ &= E_0 r - E_0 r\, tt' (-1 + (-1)^2 (r')^2 + (-1)^3 (r')^4 + \dots) \end{aligned}$$

$$E_{refl} = E_0 r \left(1 + tt' \underbrace{\left(1 - (r')^2 + (r')^4 - (r')^6 \cdots\right)}_{1/1+r^2}\right)$$

$$= E_0 r \left(1 + \frac{1-r^2}{1+r^2}\right) = E_0 \frac{2r}{1+r^2}$$

for r close to 1 → most refl.
this is max R.

General case: arbitrary Λ
let $\delta = k_0 \Lambda$

$$E_{refl} = E_0 r + E_0 tt' \left(r' e^{i\delta} + (r')^3 e^{2i\delta} + \cdots\right)$$

$r' = -r$ $$E_0 r - E_0 tt' r e^{i\delta} \left(1 + (r')^2 e^{i\delta} + (r')^4 e^{2i\delta} + \cdots\right)$$

in previous sum, $(r')^2 \to (r')^2 e^{i\delta}$

$$E_r = E_0 r - E_0 r \frac{(1-r^2)e^{i\delta}}{1-(r')^2 e^{i\delta}}$$

$$= E_0 r \left(1 - \frac{(1-r^2)e^{i\delta}}{1-r^2 e^{i\delta}}\right) = E_0 r \left(\frac{1-e^{i\delta}}{1-r^2 e^{i\delta}}\right)$$

$$r_{FP} = \frac{r(1-e^{i\delta})}{1-r^2 e^{i\delta}}$$ <u>complex</u>

next calc. I_r, I_t

$$I_r = \frac{1}{2}\varepsilon_0 n_1 c|E_r|^2 = I_0 \frac{r^2(1-e^{i\delta})(1-e^{-i\delta})}{(1+r^2e^{i\delta})(1-r^2e^{-i\delta})}$$

$$= I_0 \frac{2r^2(1-\cos\delta)}{1+r^4-2r^2\cos\delta} \equiv I_0 \cdot R$$

$\cos\delta = 1-2\sin^2\delta/2$

denom: $1+r^4-2r^2(1-2\sin^2\delta/2)$
$= (1-r^2)^2 + 4r^2\sin^2\delta/2$

$$R = \frac{2r^2 \cdot 2\sin^2\delta/2}{(1-r^2)^2+4r^2\sin^2\delta/2}$$

define $F = \left[2r/(1-r^2)\right]^2$ "coeff. of finesse"

$$R = \frac{F\sin^2\delta/2}{1+F\sin^2\delta/2}$$

$$T = 1-R = \frac{1}{1+F\sin^2\delta/2}$$

"Airy function" one of many

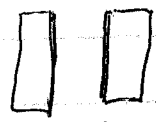
let $R_m = r^2$
$F = 4R_m/(1-R_m)^2$

also used:
$\mathcal{F} \equiv \pi\sqrt{R_m}/(1-R_m) = \pi\frac{\sqrt{F}}{2}$ "finesse"

$\delta = k_0 2nd\cos\theta_t$

Notes on F-P devices

- typical construction

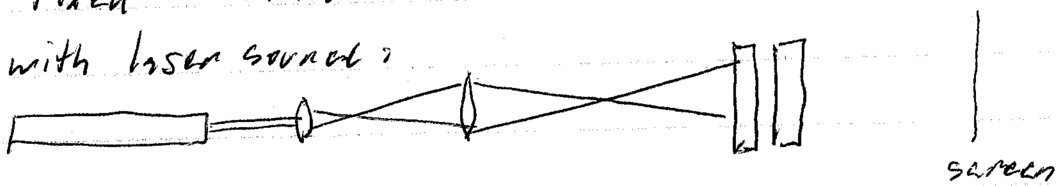


coating on inside

variable gap → "interferometer"

fixed → "étalon"

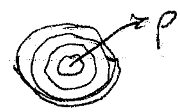
setup with laser source:



each "ray" is incident at a different angle

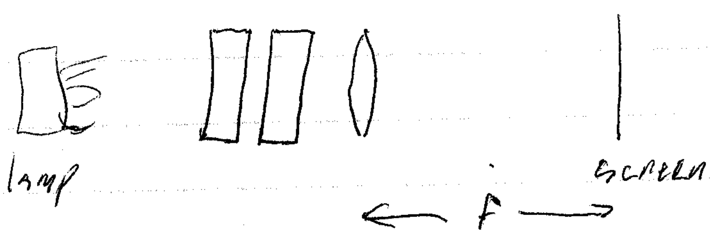
→ constructive or destructive int.

∴ we see a ring pattern



tighter spacing as ρ increases.

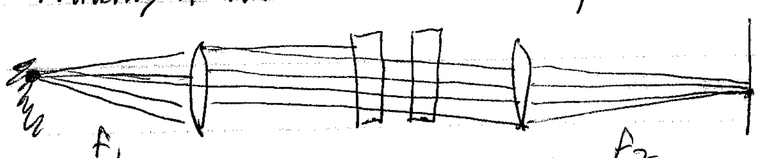
w/ extended source



each plane wave is incident at θ_i

→ focus at screen.

Adding a lens in front helps to improve fringes



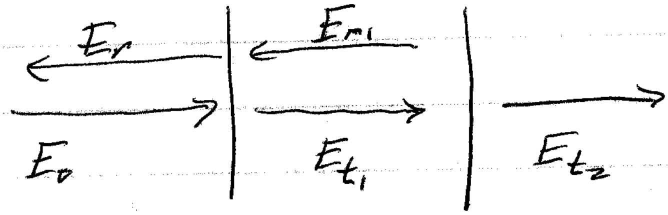
each point → collimated at particular angle.

image of lamp at screen

this reduces effect of spatial incoherence.

Compare to EM boundary value problem:

Here, assume plane waves throughout



Apply boundary cond. e.g. $E_0 + E_r = E_{t_1} + E_{r_1}$

similar for B-field

express $B \rightarrow E$

eliminate E_{r_1}, E_{t_1} $\rightarrow$ $t = E_{t_2}/E_0$

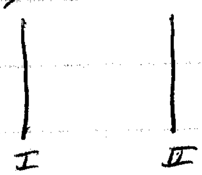
This is inherently a frequency-domain approach, steady-state

$\rightarrow r(\omega), t(\omega)$

calc impulse response to get time domain.

Thin films - multilayer

- choose fields that are continuous across interface.
  - tangential E, H
- rather than calc r, t calc E, H at boundary in terms of next boundary



$$E_I = a E_{II} + b H_{II}$$
$$H_I = c E_{II} + d H_{II}$$

$\rightarrow$ matrix form

- multiply matrices through stack.

$\rightarrow$ AR, HR coatings interference filters