

FOURIER SERIES REPRESENTATIONS OF PERIODIC FUNCTIONS

1. Consider the shifted function,

$$f(x) = x + \alpha, \quad \alpha \in \mathbb{R}, \quad (1)$$

which is  $2\pi$ -periodic on the interval  $-\pi < x < \pi$ .

- Graph  $f$  on  $(-2\pi, 2\pi)$ .
- Is the function even, odd or neither?
- Determine the Fourier coefficients  $a_0, a_n, b_n$  of  $f$ .
- Using <http://www.tutor-homework.com/grapher.html> graph the first five terms of your Fourier Series Representation of  $f$  assuming  $\alpha = 1$ .

**Hint:** First reference theorem 11.3.2 on page 492 and then recall that we found the Fourier Series of  $f(x) = x, x \in (-\pi, \pi)$  in class.

2. Consider the function,

$$f(x) = x^2, \quad x \in [-\pi, \pi], \quad (2)$$

which is  $2\pi$ -periodic on the stated interval.

- Graph  $f$  on  $(-2\pi, 2\pi)$ .
- Is the function even, odd or neither?
- Determine the Fourier coefficients  $a_0, a_n, b_n$  of  $f$ .
- Using <http://www.tutor-homework.com/grapher.html> graph the first five terms of your Fourier Series Representation of  $f$ .

**Hint:** Use symmetry arguments to simplify your calculations.

3. Consider the function,

$$f(x) = x^2, \quad x \in [0, 2\pi], \quad (3)$$

which is  $2\pi$ -periodic on the stated interval.

- Graph  $f$  on  $(-4\pi, 4\pi)$ .
- Is the function even, odd or neither?
- Determine the Fourier coefficients  $a_0, a_n, b_n$  of  $f$ .
- Using <http://www.tutor-homework.com/grapher.html> graph the first five terms of your Fourier Series Representation of  $f$ .

**Hint:** This problem can be dealt with one of two ways. One method is to write the function  $f$  down as a piecewise function from  $-\pi$  to  $\pi$  and proceed as usual. This is cumbersome and a simpler method is available. First note that the orthogonality relations, 11.1.1 page 482, HW5.P3, hold for **any**  $2\pi$  interval of integration.<sup>1</sup> If this is true then the coefficients of the Fourier series representation of  $f$  can be redeveloped for any  $2\pi$  interval.<sup>2</sup> In this way it is possible to shift the Fourier series to use the principle domain,  $x \in [0, 2\pi]$ , of the periodic function instead of the domain  $[-\pi, \pi]$ .

---

<sup>1</sup>Can you show this?

<sup>2</sup>What would the formulas look like in this case?

4. Let

$$f(x) = \begin{cases} 0, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$$

be a 4-periodic function. That is  $f(x + p) = f(x)$  where  $p = 4$ .

- (a) Graph  $f$  on  $(-4, 4)$ .
- (b) Is the function even, odd or neither?
- (c) Determine the Fourier coefficients  $a_0, a_n, b_n$  of  $f$ .
- (d) Using <http://www.tutor-homework.com/grapher.html> graph the first five terms of your Fourier Series Representation of  $f$ .

5. Let

$$u(t) = \begin{cases} 0, & -L < t < 0 \\ E\sin(\omega t), & 0 < t < L \end{cases}$$

where  $E$  represents the amplitude and  $\omega$  the frequency of the output of the half-wave rectifier. Assume that  $u(t)$  is a  $2L$ -periodic function. That is  $u(t + p) = u(t)$  where  $p = 2L$ .

- (a) Graph  $u$  on  $(-2L, 2L)$ .
- (b) Is the function even, odd or neither?
- (c) Determine the Fourier coefficients  $a_0, a_n, b_n$  of  $u$ .
- (d) Using <http://www.tutor-homework.com/grapher.html> graph the first five terms of your Fourier Series Representation of  $u$ , assuming that  $E = \omega = 1$  and  $L = \pi$ .

**Hint:** This problem is an example from section 11.2 of the book. Many steps have been omitted and the goal is to reproduce the work. You should use the example in the text and our example from class to guide your steps.