Note Title 4/23/2007

$$\underline{M} = k s^2 \hat{\varphi}$$

$$\vec{\nabla} \times \underline{M} = \vec{J}_b \qquad \vec{M} \times \hat{n} = \vec{K}_b$$

bound currents

Top view

Given $\vec{M}$ we can find currents. Currents give $\vec{B}$ through Biot & Savart or Ampere's law

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review of dielectrics

$$\sigma_b = \vec{P} \cdot \hat{n} \qquad \rho_b = -\vec{\nabla} \cdot \vec{P}$$

$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = \rho = \rho_f + \rho_b = \rho_f - \vec{\nabla} \cdot \vec{P}$$

$$\vec{\nabla} \cdot \underbrace{\left(\epsilon_0 \vec{E} + \vec{P}\right)}_{\vec{D} \text{ vector field}} = \rho_f \qquad \vec{\nabla} \cdot \vec{D} = \rho_f$$

$\oint \vec{P} \cdot d\vec{a} \neq 0$

$$\vec{\nabla} \times \vec{D} = \vec{\nabla} \times \left(\epsilon_0 \vec{E} + \vec{P}\right) = \epsilon_0 \underbrace{\vec{\nabla} \times \vec{E}}_{0} + \vec{\nabla} \times \vec{P}$$

$$\int \vec{\nabla} \times \vec{P} \cdot d\vec{a} = \oint \vec{P} \cdot d\vec{l}$$

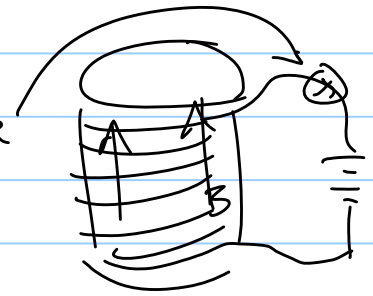
$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} = \vec{J} = \vec{J}_f + \vec{J}_b$$

$\vec{J}_b = \vec{\nabla}\times\vec{M}$

we can measure

determine J_b & atomic properties



$$\vec{\nabla}\times\underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)}_{\vec{H}} = \vec{J}_f$$

$\vec{H}$ defines $\vec{H}$

$$\vec{\nabla}\times\vec{H} = \vec{J}_f$$

or $\oint \vec{H}\cdot d\vec{l} = I_{f\,\text{enclosed}}$

To uniquely specify vector field we need $\vec{\nabla}\times$ & $\vec{\nabla}\cdot$

$$\vec{\nabla}\cdot\vec{H} = \frac{1}{\mu_0}\underbrace{\vec{\nabla}\cdot\vec{B}}_{0} - \vec{\nabla}\cdot\vec{M} \neq 0 \quad \text{in general}$$

Ex: take a bar magnet. $J_f = 0 = I_f$

$$\vec{\nabla}\times\vec{H} = 0 \quad \text{so is } \vec{H} = 0 = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\cancel{B \overset{?}{=} \mu_0 M}$$

$$\int \vec{\nabla}\cdot\vec{M}\,d\tau = \oint \vec{M}\cdot d\vec{a}$$

$\int_1 \vec{M}\cdot d\vec{a} = 0$ $\quad$ $\int_2 \vec{M}\cdot d\vec{a} = 0$ $\quad$ $\vec{M}\perp da$

$\int_3 \vec{M}\cdot d\vec{a} = M\,\text{area}$ $\quad$ $\oint M\cdot d\vec{a} \neq 0$

Need to find $\vec{M}$. Assume linear material

$$\vec{M} = \chi_m \vec{H}$$

χ_m ← mag susceptibility

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \underline{M} = \frac{\vec{B}}{\mu_0} - \chi_m \vec{H}$$

Linear

$$\underbrace{\mu_0(1+\chi_m)}_{\mu} \vec{H} = \boxed{\vec{B} = \mu \vec{H}}$$

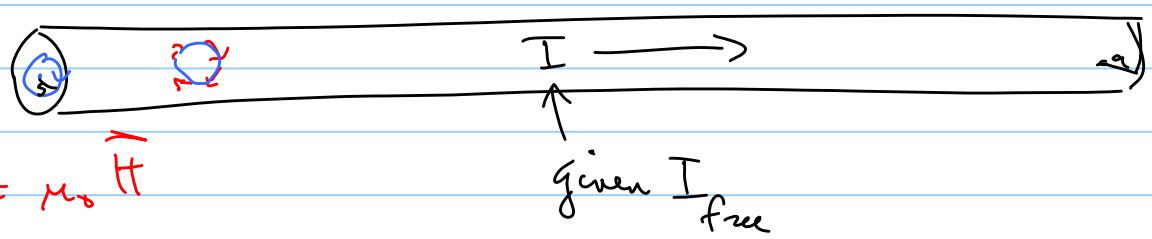
What is $\vec{J}_b$ in a Linear material

$$\vec{\nabla} \times \underset{\chi_m \vec{H}}{\underset{\parallel}{\vec{M}}} = \vec{J}_b = \chi_m \underbrace{\vec{\nabla} \times \vec{H}}_{J_f} = J_b$$

Linear material

$$\boxed{\vec{J}_b = \chi_m \vec{J}_f}$$

Uniformly distributed free current I flows down a long straight copper wire of radius a and susceptibility χm. Derive an expression for the magnetic field inside and outside the wire.



$$\vec{B} = \mu_0 \vec{H}$$

$$\oint \vec{H} \cdot d\vec{l} = \oint |\vec{H}||d\vec{l}| \cos 0 = \underline{H\, 2\pi s} = I_{fenclosed}$$

$$I_{fenclosed} = \int \underset{\frac{I}{\pi a^2}}{\underset{\parallel}{\vec{J}_{free}}} \cdot d\vec{a} = \frac{I}{\pi a^2} \int_0^s da = \underline{\frac{I}{\pi a^2} \pi s^2}$$

$$\vec{H} = \frac{I s^{\cancel{2}}}{2\pi \cancel{s} a^2} = \frac{I}{2\pi a^2} s \quad \text{inside}$$

$$\vec{B} = \mu \vec{H} = \mu \frac{I s}{2\pi a^2} \quad \text{inside}$$

$$H\, 2\pi s = I \qquad H = \frac{I}{2\pi s}$$

$$\vec{B} = \mu_0 \frac{I}{2\pi s} \quad \text{outside} \qquad \text{no } \chi \text{ outside}$$

$$\vec{J}_b = \chi_m \vec{J}_f \qquad K_b = \vec{M} \times \hat{n} = \chi_m \vec{H} \times \hat{n} = \chi_m \frac{I}{2\pi a}$$

$$\vec{J}_f = \frac{I}{\pi a^2}$$

in direction opposite to I

$$I_b = J_b \pi a^2 + K_b 2\pi a = \chi_m I - \chi_m I = 0$$