We then have

$$\mathbf{S} = \mathbf{E}_0 \times \mathbf{H}_0 \cos^2 (\mathbf{k} \cdot \mathbf{r} - \omega t) \tag{2.20}$$

for the instantaneous value of the Poynting vector. Since the average value of the cosine squared is just $\frac{1}{2}$, then for the average value of the Poynting vector, we can write

$$\langle \mathbf{S} \rangle = \tfrac{1}{2} \mathbf{E}_0 \times \mathbf{H}_0 \tag{2.21}$$

(If the complex exponential form of the wave functions for **E** and **H** is used, the average Poynting flux can be expressed as $\frac{1}{2} \mathbf{E}_0 \times \mathbf{H}_0^*$. See Problem 2.4.)

Since the wave vector **k** is perpendicular to both **E** and **H**, it has the same direction as the Poynting vector. Consequently, an alternative expression for the average Poynting flux is

$$\langle \mathbf{S} \rangle = I \frac{\mathbf{k}}{k} = I \, \hat{\mathbf{n}} \tag{2.22}$$

in which $\hat{\mathbf{n}}$ is a unit vector in the direction of propagation and I is the magnitude of the average Poynting flux. The quantity I is called the *irradiance*.[2] It is given by

$$I = \frac{1}{2} E_0 H_0 = \frac{n}{2 Z_0} |E_0|^2 \tag{2.23}$$

The last step follows from the relations between the magnitudes of the electric and magnetic vectors developed in the previous section. Thus the rate of flow of energy is proportional to the square of the amplitude of the electric field. In isotropic media the direction of the energy flow is specified by the direction of **S** and is the same as the direction of the wave vector **k.** (In nonisotropic media, for example crystals, **S** and **k** are not always in the same direction. This will be discussed later in Chapter 6.)

2.3 Linear Polarization

Consider a plane harmonic electromagnetic wave for which the fields **E** and **H** are given by the expressions

$$\mathbf{E} = \mathbf{E}_0 \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t) \tag{2.24}$$

$$\mathbf{H} = \mathbf{H}_0 \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t) \tag{2.25}$$

If the amplitudes $\mathbf{E}_0$ and $\mathbf{H}_0$ are constant real vectors, the wave is said to be *linearly polarized* or plane polarized. We know from the theory

[2] Sometimes the word *intensity* is used for I, but this is not technically correct (see Chapter 7).

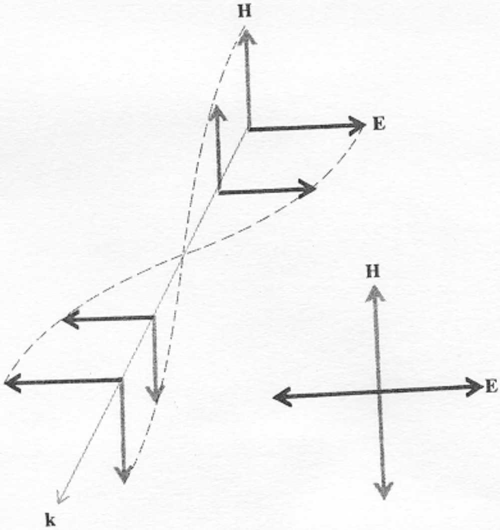

Figure 2.2. Fields in a plane wave, linearly polarized.

of the previous section that the fields **E** and **H** are mutually perpendicular. It is traditional in optics to designate the direction of the electric field as the direction of polarization. Figure 2.2 shows a diagram of the fields in a plane, linearly polarized wave.

In the case of natural, or so-called unpolarized, light the instantaneous polarization fluctuates rapidly in a random manner. A *linear polarizer* is a device that produces linearly polarized light from unpolarized light. There are several kinds of linear polarizers. The most efficient ones are those that are based on the principle of double refraction, to be treated in Chapter 6. Another type makes use of the phenomenon of anisotropic optical absorption, or *dichroism*, which means that one component of polarization is more strongly absorbed than the other. The natural crystal tourmaline is dichroic and can be used to make a polarizer, although it is not very efficient. A familiar commercial product is Polaroid, developed by Edwin Land. It consists of a thin layer of parallel needlelike crystals that are highly dichroic. The layer is embedded in a plastic sheet which can be cut and bent.

The *transmission axis* of such a polarizer defines the direction of the electric field vector for a light wave that is transmitted with little or no loss. A light wave whose electric vector is at right angles to the transmission axis is absorbed or attenuated. An ideal polarizer is one that is completely transparent to light linearly polarized in the direction of the transmission axis, and completely opaque to light polarized in the orthogonal direction to the transmission axis.

Consider the case of unpolarized light incident on an ideal linear polarizer. Now the instantaneous electric field **E** can always be resolved into two mutually perpendicular components, $\mathbf{E}_1$ and $\mathbf{E}_2$, (Figure 2.3), where $\mathbf{E}_1$ is along the transmission axis of the polarizer.

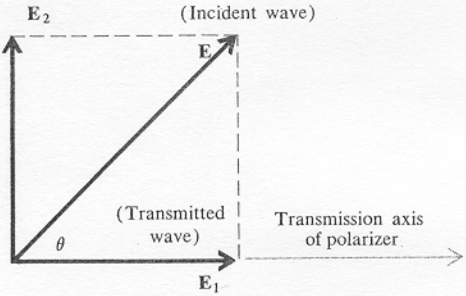

Figure 2.3. Relationship between the incident and the transmitted fields for a linear polarizer.

If **E** makes an angle θ with the transmission axis, then the magnitude of the transmitted field is

$$E_1 = E \cos\theta$$

The transmitted intensity I_1, being proportional to the square of the field, is therefore given by

$$I_1 = I \cos^2\theta$$

where I is the intensity of the incident beam. For unpolarized light all values of θ occur with equal probability. Therefore, the transmission factor of an ideal linear polarizer for unpolarized light is just the average value of $\cos^2\theta$ namely, $\frac{1}{2}$.

Partial Polarization Light that is partially polarized can be considered to be a mixture of polarized and unpolarized light. The *degree of polarization* in this case is defined as the fraction of the total intensity that is polarized:

$$P = \text{degree of polarization} = \frac{I_{\text{pol}}}{I_{\text{pol}} + I_{\text{unpol}}} \qquad (2.26)$$

It is left as an exercise to show that for partial linear polarization

$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{2.27}$$

where $I_{\max}$ and I_{min} refer to the intensity of the light transmitted through a linear polarizer when it is turned through the complete range of 360 degrees.

Scattering and Polarization When light propagates through a medium other than a vacuum, the electric field of the light wave induces oscillating electric dipoles in the constituent atoms and molecules of the medium. It is these induced dipoles that are mainly responsible for the optical properties of a given substance, that is, refraction, absorption, and so on. This subject will be treated later in Chapter 6. In addition to affecting the propagation of light waves, the induced dipoles can also scatter the light in various directions. This molecular scattering (as distinguished from scattering by suspended particles such as dust) was investigated by Lord Rayleigh who showed that, theoretically, the fraction of light scattered by gas molecules should be proportional to the fourth power of the light frequency or, equivalently, to the inverse fourth power of the wavelength. This accounts for the blue color of the sky since the shorter wavelengths (blue region of the spectrum) are scattered more than the longer wavelengths (red region).[3]

In addition to the wavelength dependence of light scattering, there is also a polarization effect. This comes about from the directional radiation pattern of an oscillating electric dipole. The maximum radiation is emitted at right angles to the dipole axis, and no radiation is emitted along the direction of the axis. Furthermore the radiation is linearly polarized along the direction of the dipole axis. Consider the case of light that is scattered through an angle of 90 degrees. The electric vector of the scattered wave will be at right angles to the direction of the incident wave, as shown in Figure 2.4, and so the scattered light is linearly polarized. The polarization of the light of the blue sky is easily observed with a piece of Polaroid. The maximum amount of polarization is found in a direction 90 degrees from the direction of the sun. Measurements show that the degree of polarization can be greater than 50 percent.

2.4 Circular and Elliptic Polarization

Let us return temporarily to the real representation for electromagnetic waves. Consider the special case of two linearly polarized

[3] Actually, the sky would appear violet rather than blue were it not for the fact that the color sensitivity of the eye drops off sharply at the violet end of the spectrum and also that the energy in the solar spectrum diminishes there.

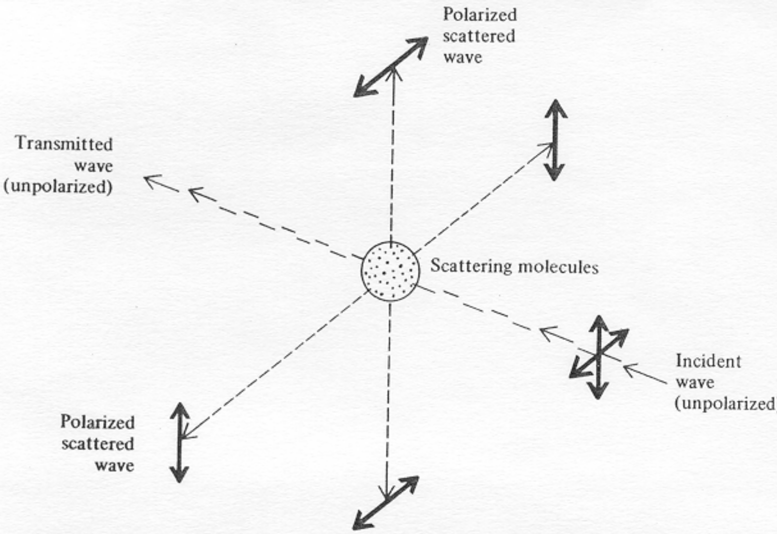

Figure 2.4. Illustrating polarization in the molecular scattering of light. The **E** vectors for the incident and scattered waves are indicated.

waves of the same amplitude E_0 polarized orthogonally to each other. Further, suppose the waves have a phase difference of $\pi/2$. We choose coordinate axes such that the electric vectors of the two waves are in the x and y directions, respectively. Accordingly, the component electric fields are

$$\hat{\mathbf{i}}E_0 \cos(kz - \omega t)$$

$$\hat{\mathbf{j}}E_0 \sin(kz - \omega t)$$

The total electric field **E** is the vector sum of the two component fields, namely,

$$\mathbf{E} = E_0 \left[\hat{\mathbf{i}} \cos(kz - \omega t) + \hat{\mathbf{j}} \sin(kz - \omega t)\right] \tag{2.28}$$

Now the above expression is a perfectly good solution of the wave equation. It can be interpreted as a single wave in which the electric vector at a given point is constant in magnitude but rotates with angular frequency ω. This type of wave is said to be *circularly polarized*. A drawing showing the electric field and associated magnetic field of circularly polarized waves is shown in Figure 2.5.

The signs of the terms in Equation (2.28) are such that the expression represents *clockwise* rotation of the electric vector *at a given point in space* when viewed against the direction of propagation. Also, *at a given instant in time*, the field vectors describe right-

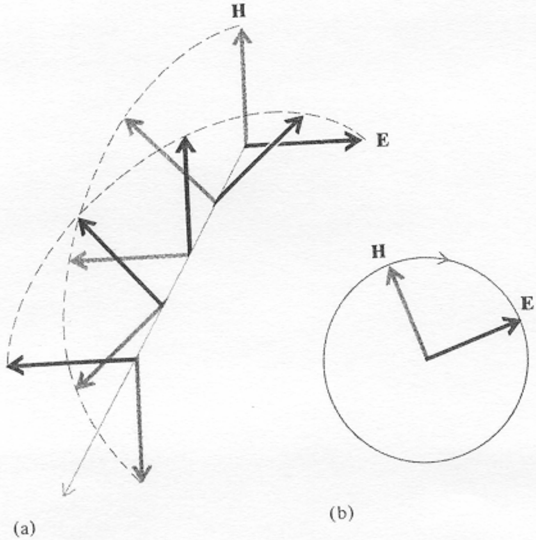

Figure 2.5. Electric and magnetic vectors for right circularly polarized light. (a) Vectors at a given instant in time; (b) rotation of the vectors at a given position in space.

handed spirals as illustrated in Figure 2.5. Such a wave is said to be *right circularly polarized.*

If the sign of the second term is changed, then the sense of rotation is changed. In this case the rotation is counterclockwise at a given point in space when viewed against the direction of propagation, and, at a given instant in time the fields describe left-handed spirals. The wave is then called *left circularly polarized.*

It should perhaps be pointed out here that if one "rides along" with the wave, then the field vectors do not change in either direction or magnitude, because the quantity $kz-\omega t$ remains constant. This is true for any type of polarization.

Let us now return to the complex notation. The electric field for a circularly polarized wave can be written in complex form as

$$\mathbf{E} = \hat{\mathbf{i}}E_0 \exp i(kz - \omega t) + \hat{\mathbf{j}}E_0 \exp i(kz - \omega t \pm \pi/2) \tag{2.29}$$

or, by employing the identity $e^{i\pi/2} = i$, we can write

$$\mathbf{E} = E_0(\hat{\mathbf{i}} \pm i\hat{\mathbf{j}}) \exp i(kz - \omega t) \tag{2.30}$$

It is easy to verify that the real part of the above expression is precisely that of Equation (2.28) where, however, the minus sign must be used to represent right circular polarization and the plus sign for left circular polarization.

The reader is reminded here that if one uses the wave function $\exp i(\omega t - kz)$ rather than $\exp i(kz - \omega t)$, then the opposite sign convention applies.

Elliptic Polarization If the component (real) fields are not of the same amplitude, say $\hat{\mathbf{i}}E_0 \cos(kz - \omega t)$ and $\hat{\mathbf{j}}E_0' \sin(kz - \omega t)$ where $E_0 \neq E_0'$, the resultant electric vector, at a given point in space, rotates and also changes in magnitude in such a manner that the end of the vector describes an ellipse as illustrated in Figure 2.6. In this case the wave is said to be *elliptically polarized.*

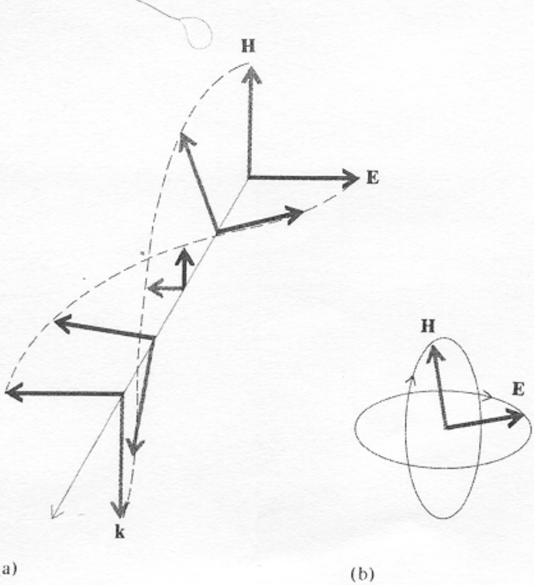

Figure 2.6. Electric and magnetic vectors for right elliptically polarized light. (a) Vectors at a given instant in time; (b) at a given position in space.

It is sometimes convenient to employ a *complex vector amplitude* $\mathbf{E}_0$ defined as follows:

$$\mathbf{E}_0 = \hat{\mathbf{i}}E_0 + i\hat{\mathbf{j}}E_0' \tag{2.31}$$

The corresponding wave function is

$$\mathbf{E} = \mathbf{E}_0 \exp i(kz - \omega t) \tag{2.32}$$

This expression can represent any type of polarization. Thus if $\mathbf{E}_0$ is real, we have linear polarization, whereas if it is complex, we have elliptic polarization. In the special case of circular polarization the real and imaginary parts of $\mathbf{E}_0$ are equal.

Quarter-Wave Plate Circularly polarized light can be produced by introducing a phase shift of $\pi/2$ between two orthogonal components of linearly polarized light. One device for doing this is known as a *quarter-wave plate*. These plates are made of *doubly refracting* transparent crystals, such as calcite or mica.[4] Doubly refracting crystals have the property that the index of refraction differs for different directions of polarization. It is possible to cut a doubly refracting crystal into slabs in such a way that an axis of maximum index n_1 (the slow axis) and an axis of minimum index n_2 (the fast axis) both lie at right angles to one another in the plane of the slab. If the slab thickness is d, then the optical thickness is $n_1 d$ for light polarized in the direction of the slow axis and $n_2 d$ for light polarized in the direc-

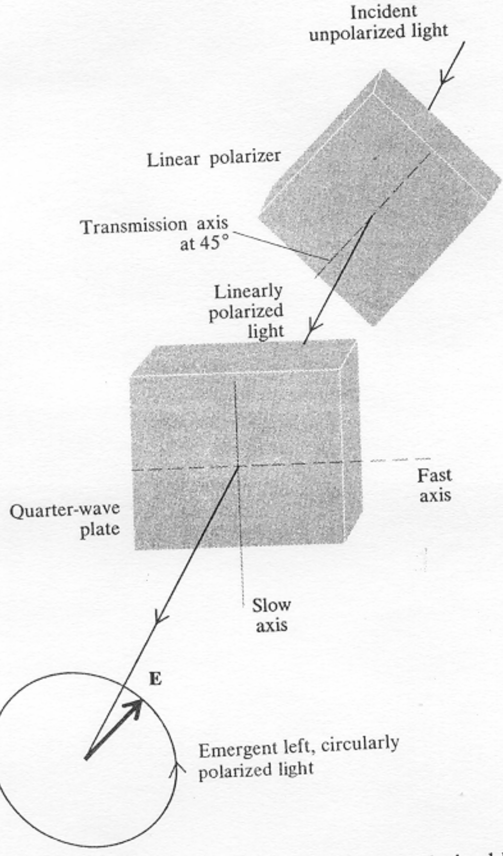

Figure 2.7. Arrangement for producing circularly polarized light.

[4] The optics of crystals will be treated in detail in Chapter 6.

tion of the fast axis. For a quarter-wave plate, d is chosen to make the difference $n_1 d - n_2 d$ equal to one-quarter wavelength, so that d is given by the equation

$$d = \frac{\lambda_0}{4(n_1 - n_2)} \tag{2.33}$$

in which λ_0 is the vacuum wavelength.

The physical arrangement for producing circularly polarized light is shown in Figure 2.7. Incident unpolarized light is made linearly polarized by means of a linear polarizer such as a sheet of Polaroid. The quarter-wave plate is placed in the beam of linearly polarized light. The orientation of the quarter-wave plate is defined by the angle θ between the transmission axis of the Polaroid and the fast axis of the quarter-wave plate. By choosing θ to be 45 degrees, the light entering the quarter-wave plate can be resolved into two orthogonal linearly polarized components of equal amplitude and equal phase. On emerging from the quarter-wave plate, these two components are out of phase by $\pi/2$. Hence the emerging light is circularly polarized.

The sense of rotation of the circularly polarized light depends on the value of θ and can be reversed by rotating the quarter-wave plate through an angle of 90 degrees so that θ is 135 degrees. If θ is any value other than ± 45 degrees or ± 135 degrees, the polarization of the emerging light will be elliptic rather than circular.

2.5 Matrix Representation of Polarization. The Jones Calculus

The complex vector amplitude given in the preceding section, Equation (2.31), is not the most general expression, because it was assumed that the x component was real and the y component was imaginary. A more general way of expressing the complex amplitude of a plane harmonic wave is

$$\mathbf{E}_0 = \hat{\mathbf{i}} E_{0x} + \hat{\mathbf{j}} E_{0y} \tag{2.34}$$

where E_{0x} and E_{0y} can *both* be complex. Accordingly, they can be written in exponential form as

$$E_{0x} = |E_{0x}|\, e^{i\phi_x} \tag{2.35}$$

$$E_{0y} = |E_{0y}|\, e^{i\phi_y} \tag{2.36}$$

A convenient notation for the above pair of complex amplitudes is the following matrix known as the *Jones vector:*

$$\begin{bmatrix} E_{0x} \\ E_{0y} \end{bmatrix} = \begin{bmatrix} |E_{0x}|\, e^{i\phi_x} \\ |E_{0y}|\, e^{i\phi_y} \end{bmatrix} \tag{2.37}$$

The *normalized* form of the Jones vector is obtained by dividing by the square root of the sum of the squares of the two moduli, namely $(|E_{0x}|^2 + |E_{0y}|^2)^{1/2}$. A useful but not necessarily normalized form is obtained by factoring out any common factor that results in the simplest expression. For example, $\begin{bmatrix} A \\ 0 \end{bmatrix} = A \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ represents a wave linearly polarized in the x direction, and $\begin{bmatrix} 0 \\ A \end{bmatrix} = A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ a wave linearly polarized in the y direction. The vector $\begin{bmatrix} A \\ A \end{bmatrix} = A \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ represents a wave that is linearly polarized at 45 degrees relative to the x axis. Circular polarization is represented by $\begin{bmatrix} 1 \\ i \end{bmatrix}$ for left circular polarization, and $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ for right circular polarization.

One of the applications of the Jones notation is calculating the result of adding two or more waves of given polarizations. The result is obtained simply by adding the Jones vectors. As an example, suppose we want to know the result of adding two waves of equal amplitude, one being right circularly polarized, the other left circularly polarized. The calculation by means of the Jones vectors proceeds as follows:

$$\begin{bmatrix} 1 \\ -i \end{bmatrix} + \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1+1 \\ -i+i \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The last expression shows that the resultant wave is linearly polarized in the x direction and its amplitude is twice that of either of the circular components.

Another use of the matrix notation is that of computing the effect of inserting a linear optical element, or a train of such elements, into a beam of light of given polarization. The optical elements are represented by 2×2 matrices called *Jones matrices*. The types of optical devices that can be so represented include linear polarizers, circular polarizers, phase retarders (quarter-wave plates, and so forth), isotropic phase changers, and isotropic absorbers. We give, without proof, the matrices for several optical elements in Table 2.1 [39].

The matrices are used as follows. Let the vector of the incident light be $\begin{bmatrix} A \\ B \end{bmatrix}$ and the vector of the emerging light be $\begin{bmatrix} A' \\ B' \end{bmatrix}$. Then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A' \\ B' \end{bmatrix} \tag{2.38}$$

where $\begin{bmatrix} ab \\ cd \end{bmatrix}$ is the Jones matrix of the optical element. If light is sent

Table 2.1. JONES MATRICES FOR SOME LINEAR OPTICAL ELEMENTS

Optical Element		Jones Matrix
Linear polarizer	Transmission axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
	Transmission axis vertical	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
	Transmission axis at ±45°	$\frac{1}{2}\begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix}$
Quarter-wave plate	Fast axis vertical	$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
	Fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
	Fast axis at ±45°	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \pm i \\ \pm i & 1 \end{bmatrix}$
Half-wave plate	Fast axis either vertical or horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Isotropic phase retarder		$\begin{bmatrix} e^{i\phi} & 0 \\ 0 & e^{i\phi} \end{bmatrix}$
Relative phase changer		$\begin{bmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{bmatrix}$
Circular polarizer	Right	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
	Left	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$

Note: Normalization factors are included in the table. These factors are necessary for energy considerations only and can be omitted in calculations concerned primarily with type of polarization. Also, the signs of all matrix elements containing the factor i should be changed if one uses the wave function exp $i(\omega t - kz)$ rather than exp $i(kz - \omega t)$.

through a train of optical elements, then the result is given by matrix multiplication:

$$\begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \cdots \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A' \\ B' \end{bmatrix} \quad (2.39)$$

To illustrate, suppose a quarter-wave plate is inserted into a beam of linearly polarized light as shown in Figure 2.6. Here the incoming beam is polarized at 45 degrees with respect to the horizontal (x axis), so that its vector, aside from an amplitude factor, is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. From the table, the Jones matrix for a quarter-wave plate with the fast-axis horizontal is $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$. The vector of the emerging beam is then given by

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

The emergent light is therefore left circularly polarized.

It should be noted that the Jones calculus is of use only for computing results with light that is initially polarized in some way. There is no Jones vector representation for unpolarized light.

Orthogonal Polarization Two waves whose states of polarization are represented by the complex vector amplitudes $\mathbf{E}_1$ and $\mathbf{E}_2$ are said to be *orthogonally polarized* if

$$\mathbf{E}_1 \cdot \mathbf{E}_2^* = 0$$

where the asterisk denotes the complex conjugate.

For linearly polarized light, orthogonality merely means that the fields are polarized at right angles to one another. In the case of circular polarization it is readily seen that right circular and left circular polarizations are mutually orthogonal states. But, there is a corresponding orthogonal polarization for any type of polarization.

In terms of Jones vectors it is easy to verify that $\begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$ and $\begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$ are orthogonal if

$$A_1A_2^* + B_1B_2^* = 0 \tag{2.40}$$

Thus, for example, $\begin{bmatrix} 2 \\ i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2i \end{bmatrix}$ represent a particular pair of orthogonal states of elliptic polarization. These are shown in Figure 2.8.

It is instructive to note that light of arbitrary polarization can always be resolved into two orthogonal components. Thus resolution into linear components is written

$$\begin{bmatrix} A \\ B \end{bmatrix} = A\begin{bmatrix} 1 \\ 0 \end{bmatrix} + B\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and into circular components is written

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{2}(A + iB)\begin{bmatrix} 1 \\ -i \end{bmatrix} + \frac{1}{2}(A - iB)\begin{bmatrix} 1 \\ i \end{bmatrix}$$

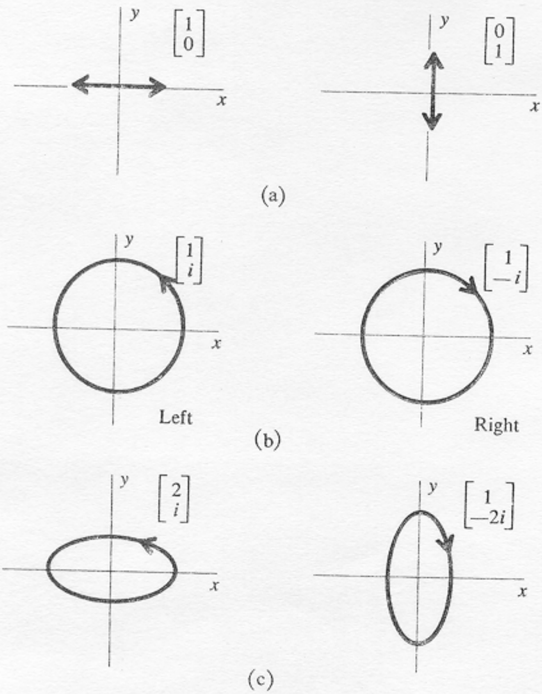

Figure 2.8. Illustrations of some Jones vectors.

Eigenvectors of Jones Matrices. An *eigenvector* of any matrix is defined as a particular vector which, when multiplied by the matrix, gives the same vector within a constant factor. In the Jones calculus this can be written

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \lambda \begin{bmatrix} A \\ B \end{bmatrix}$$

The constant λ, which may be real or complex, is called the *eigenvalue*.

Physically, an eigenvector of a given Jones matrix represents a particular polarization of a wave which, upon passing through the optical element in question, emerges with the same polarization as when it entered. However, depending on the value of λ, the amplitude and the phase may change. If we write $\lambda = |\lambda| e^{i\psi}$, then $|\lambda|$ is the amplitude change, and ψ is the phase change.

The problem of finding the eigenvalues and the corresponding eigenvectors of a 2×2 matrix is quite simple. The matrix equation above can be written as

$$\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0 \tag{2.41}$$

Now in order that a nontrivial solution exists, namely one in which A and B are not both zero, the determinant of the matrix must vanish

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0 \tag{2.42}$$

This is a quadratic equation in λ, known as the *secular equation.* Upon expanding the determinant we get

$$(a-\lambda)(d-\lambda) - bc = 0$$

whose roots λ_1 and λ_2 are the eigenvalues. To each root there is a corresponding eigenvector. These can be found by noting that the matrix equation (2.41) is equivalent to the two algebraic equations

$$(a-\lambda)A + bB = 0 \qquad cA + (d-\lambda)B = 0 \tag{2.43}$$

The ratio of A to B, corresponding to a given eigenvalue of λ, can be found by substitution of λ_1 or λ_2 into either equation.

For example, from the table of Jones matrices, a quarter-wave plate with fast-axis horizontal has the Jones matrix $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$. The secular equation is then

$$(1-\lambda)(i-\lambda) = 0$$

which gives $\lambda = 1$ and $\lambda = i$ for the two eigenvalues. Equations (2.43) then read $(1-\lambda)A = 0$ and $(i-\lambda)B = 0$. Thus, for $\lambda = 1$ we must have $A \neq 0$ and $B = 0$. Similarly, for $\lambda = i$ it is necessary that $A = 0$ and $B \neq 0$. Hence the normalized eigenvectors are $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ for $\lambda = 1$, and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for $\lambda = i$. Physically, the result means that light that is linearly polarized in the direction of either the fast axis or the slow axis is transmitted without change of polarization. There is no change in amplitude since $|\lambda| = 1$ for both cases, but a relative phase change of $\pi/2$ occurs since $\lambda_2/\lambda_1 = i = e^{i\pi/2}$.

2.6 Reflection and Refraction at a Plane Boundary

We now investigate the very basic phenomena of reflection and refraction of light from the standpoint of electromagnetic theory. It is assumed that the reader is already familiar with the elementary rules of reflection and refraction and how they are deduced from Huygens'