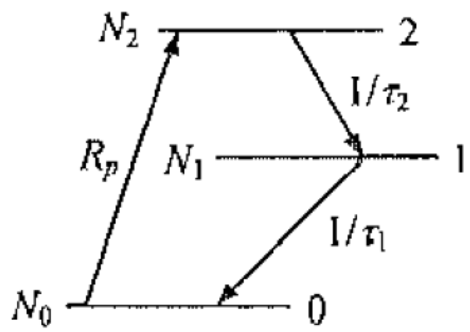
PHGN 480: Laser Physics due Wednesday 23 Oct 2013
Homework 4

1) Consider the energy level scheme for a 3-level laser shown in the figure below. Starting at t = 0, where the atoms are in the ground state, the atoms are raised from level 0 to level 2 at a pump rate R_p. The lifetime of levels 1 and 2 are τ_1 and τ_2 respectively. Assuming that the ground state 0 is not depleted to any significant extent and neglecting stimulated emission:
   a. Write the rate equations for the population densities, N_1 and N_2, of level 1 and 2 respectively;
   b. Analytically calculate N_1 and N_2 as a function of time. If you want to check your work, you might use DSolve[ ] in Mathematica, but show the steps how to get the solution.
   c. Use Mathematica to plot the population densities (N_1(t) and N_2(t)) and the population inversion (N_2(t) - N_1(t)) on the same plot. Make two plots using the following two input values (you may pick an arbitrary value of R_p):
      i. $\tau_1 = 2$ μs, $\tau_2 = 1$ μs
      ii. $\tau_1 = 1$ μs, $\tau_2 = 2$ μs
   d. Is it possible to achieve a population inversion for each of the two cases? If so, what are the requirements on the pump pulse?

   Hint: the differential equation for the population of level 1 can be solved by multiplying both sides by the factor exp(t/τ_1). In this way the left-hand side of the preceding differential equation becomes a perfect differential.



2) **A more general derivation of the saturation in a two level system.** In this version, we have pump rates (/vol/time) R_1 and R_2, and overall lifetimes out of the levels τ_1 and τ_2. We also include the effect of degeneracies on levels 1 and 2, g_1 and g_2. We'll work with the beam intensity and the cross-section rather than the pump rate $W = \frac{I\sigma_{21}}{h\nu_L}$. With these included, the rate equations are

$$\frac{dN_2}{dt} = R_2 - \Delta N^* \sigma_{21} \frac{I}{h\nu_L} - \frac{N_2}{\tau_2}$$

$$\frac{dN_1}{dt} = R_1 + \Delta N^* \sigma_{21} \frac{I}{h\nu_L} - \frac{N_1}{\tau_1} + N_2 A_{21}$$

where

$$\Delta N^* = N_2 - \frac{g_2}{g_1} N_1$$

is the population inversion density. Remember that in this formulation, spontaneous emission out of level 2 is included in the lifetime in that level, τ_2.

a. (extra credit) By setting the time derivatives = 0 for steady state, you can calculate the steady state expressions for N_1^{ss} and N_2^{ss}, then calculate the steady-state value of ΔN^*, to show that

$$\Delta N^*(I) = N_2^{ss} - \frac{g_2}{g_1} N_1^{ss} = \frac{R_2\tau_2\left[1-(g_2/g_1)A_{21}\tau_1\right]-(g_2/g_1)R_1\tau_1}{1+\sigma_{21} I \frac{1}{h\nu_L}\left[\tau_2+(g_2/g_1)\tau_1-(g_2/g_1)A_{21}\tau_1\tau_2\right]}$$

b. (required, not extra credit) This equation can be written in the standard form $\Delta N^*(I) = \frac{\Delta N^*(0)}{1+I/I_s}$, with the saturation intensity $I_s = \frac{h\nu_L}{\sigma_{21}\tau_R}$.

Write expressions for $\Delta N^*(0)$ and the "recovery time" τ_R and give a description of what each of those terms mean physically.

3) A laser-pumped amplifier, pumped longitudinally, has an inversion density distribution

$$N^*(r,z) = N_0^* \exp\left[-2r^2/w_p^2 - \kappa z\right]$$

a. Let the total stored (i.e. extractable) energy be E_{stor}, the crystal length is L, and the photon energy is $\hbar\omega_L$, calculate an expression for the peak inversion density N_0^* by integrating over the volume of the crystal.

b. Now consider the pumping process to create this inversion density. Let the pump energy be $E_0 = 50\text{mJ}$, crystal length be L = 1 cm, pump 1/e^2 radius be $w_p = 2\text{mm}$, and the absorption coefficient be κ = 3/cm. The pump wavelength is 532nm and the laser wavelength is 800nm. Calculate the peak inversion density N_0^* in cm^{-3}.

c. The small-signal single-pass gain in the amplifier is ultimately independent of the axial (z) variation of the inversion density. The small signal gain can be calculated as $G(r) = \exp\left[\sigma_{21}\int_0^L N^*(r,z)dz\right] = \exp\left[\Gamma_{stor}(r)/\Gamma_s\right]$, where $\Gamma_{stor}(r)$ is the called stored energy fluence and $\Gamma_s = \hbar\omega_L/\sigma_{21}$ is the saturation fluence. Using the inversion density calculated in part b, calculate $\Gamma_{stor}(r)$.

d. If our input seed pulse (to be amplified) has an energy of E_{in}, the input energy fluence distribution is $\Gamma(r) = \Gamma_0 \exp\left[-2r^2 / w_L^2\right]$, with Γ_0 calculated so that integrating over the beam area gives E_{in}. On one graph, plot the input and output energy fluence for w_L = 5mm and for w_L = 1mm. To see how the beam is reshaped by the gain, normalize each curve so that the peak is = 1. The phenomenon you are observing is spatial gain narrowing. Since you are calculating small signal gain (without saturation), that absolute values of the input energy and the saturation fluence are not important – you may assume any value for these parameters.