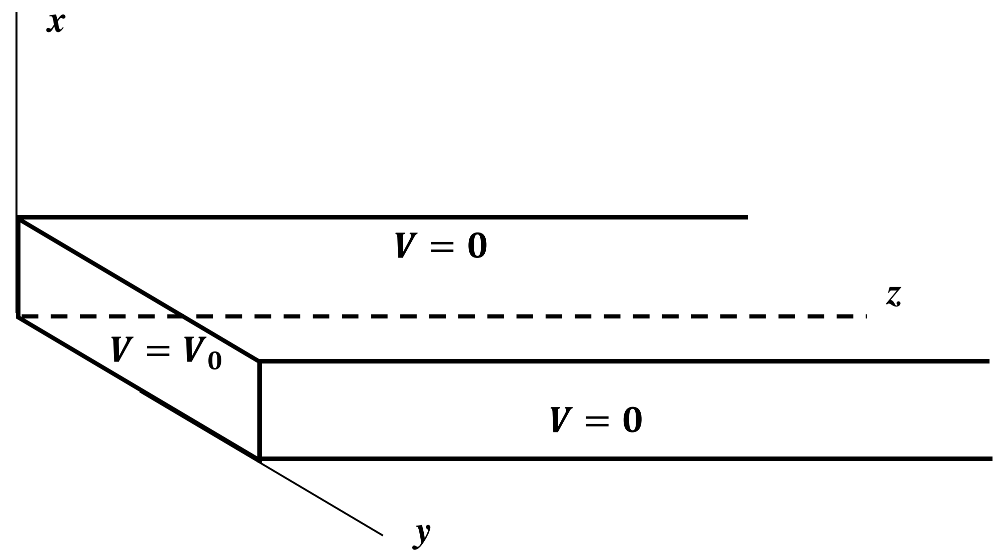
PHGN 361 Recitation 1 – Spring 2016

We have a rectangular, hollow conducting pipe of length L in the y direction and width W in the x direction. The pipe is capped at $z = 0$, and then extends infinitely in the positive z direction. The cap is being held at some voltage V_0, and the other four sides of the pipe are grounded ($V = 0$).



We're going to find the potential everywhere inside the pipe using separation of variables.

Getting started & separating variables

1a) Start by sketching the problem on a sheet of paper, and write down Laplace's equation in Cartesian coordinates. Why are we solving Laplace's equation instead of Poisson's equation?

b) Can we exclude any of the variables using symmetry, or do we have to attend to all of x, y, and z?

c) Execute the actual separation of variables steps explicitly. Guess a product solution, shove it into Laplace's equation, and manipulate appropriately until you obtain single-variable terms.

d) Argue convincingly that what you have is actually separate ordinary differential equations, each equal to some constant. Explicitly note the relationship between the different separation constants.

Matching boundary conditions qualitatively

2a) List all the different kinds of functions you *might* be able to use to construct a solution to our differential equations, without worrying yet about our specific boundary conditions.

b) Now look at each variable in turn and decide what kind of function you can use to match the boundary conditions in that direction. Make sure that the choices you're making respect the fact that the individual separation constants must add up to zero.

c) Smoosh all your single-variable functions together into one big product solution. Write it as an infinite sum to represent the fact that there may be many terms in the final answer. You should have some arbitrary constants here and there. Some will be part of the arguments of your functions, and others will be out in front. Is it a single sum, a double sum, or a triple sum?

Matching boundary conditions quantitatively

3a) Use the boundary conditions in x and y to fix the constant(s) in the arguments of your functions.

b) Use the boundary condition at $z = 0$ to fix the constant(s) in front of your functions.

c) Write out the complete solution for the potential everywhere inside the pipe.

Reflection

4a) Spend a minute thinking about the form of the solution and see if it makes sense to. Write down a sentence or two on the matter.

b) You should have an answer in the form of an infinite series, but it's possible to do something pretty neat. Look at the form of the series and see if you can convince yourself that it converges *really* fast. Like, possibly so fast that just using the first term in the series might give you pretty good accuracy.

c) Open up Mathematica and see if you can get it to produce a nice plot of your solution. Since it'll be a three variable function, you might have to think a bit about how best to represent it. Do as you please.