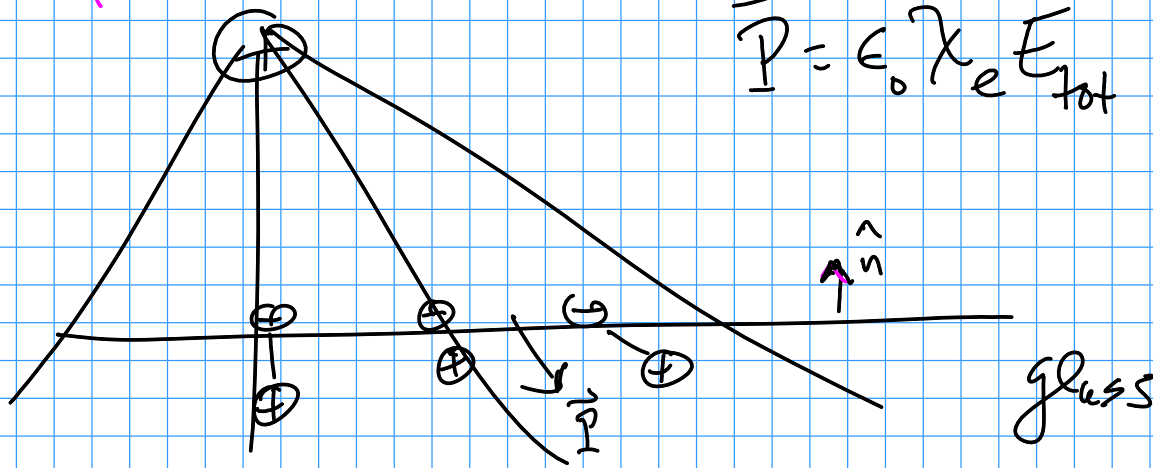
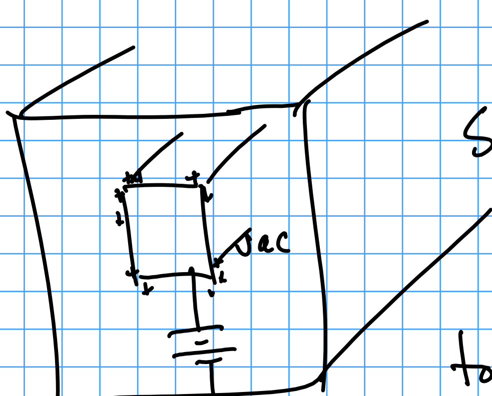
$$\vec{P} = \epsilon_0 \chi_e \vec{E}_{tot}$$



We want to find bound charge & then use all these charges to determine $\vec{E}$ everywhere

σ_b & pt charge

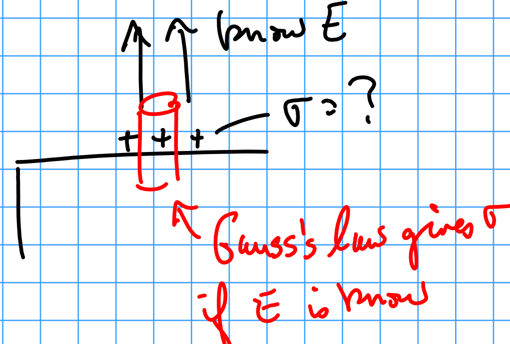
$$\sigma_b = \vec{P}\cdot\hat{n} = P^{\perp} = P_z = \epsilon_0 \underset{\text{linear}}{\chi} E^{\perp}_{tot} = \sigma_b$$



Solved Laplace's eqn to get V

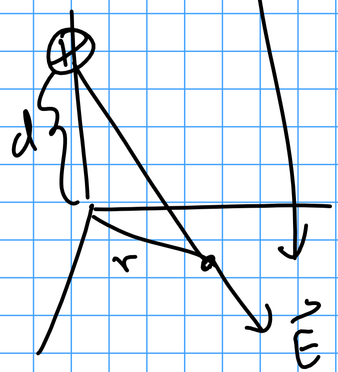
$$\vec{E} = -\vec{\nabla} V$$

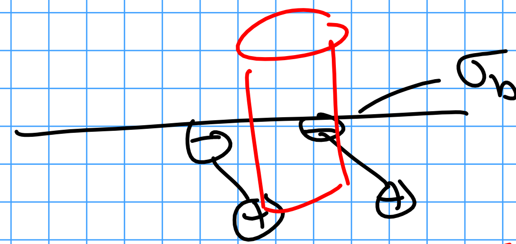
to get σ



$E_{pt}^{\perp}$

$$E_{tot}^{\perp} = E_{pt.\ charge}^{\perp} + E_{bound\ charge}^{\perp} = -\frac{1}{4\pi\epsilon_0}\frac{qd}{(r^2+d^2)^{3/2}} +$$



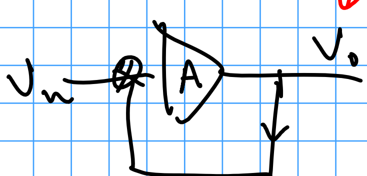


Gauss's Law $\Rightarrow$ $E_{bound}^{\perp} = \frac{\sigma_b}{2\epsilon_0}$

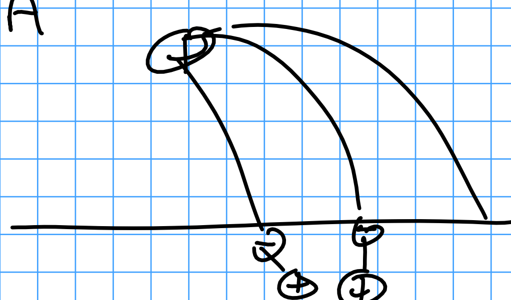
$$E_{tot}^{\perp} = -\frac{1}{4\pi\epsilon_0}\frac{qd}{(r^2+d^2)^{3/2}} + \frac{\sigma_b}{2\epsilon_0}$$

$$\epsilon_0\chi_e E_{tot}^{\perp} = \sigma_b = \epsilon_0\chi_e\left\{-\frac{1}{4\pi\epsilon_0}\frac{qd}{(r^2+d^2)^{3/2}} + \frac{\sigma_b}{2\epsilon_0}\right\}$$

$V_{out} = AV_{in}$



$V_0 = AV_{in} + V_0 A$



Cons. charge

$$\vec{J} = \underset{\uparrow\ C/m^3}{\rho}\, \overset{m/s\ \swarrow}{\vec{v}}$$

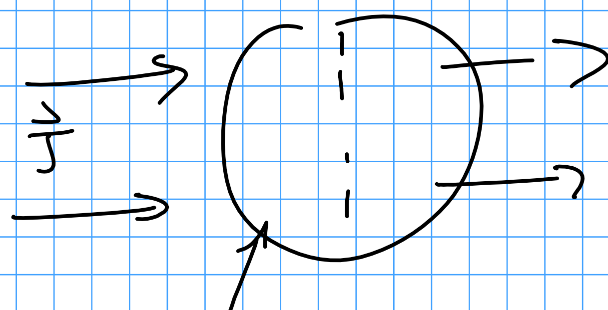
$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

$\uparrow$ dynamic

static $\quad \vec{\nabla}\cdot\vec{J} = 0 \qquad \left(\begin{array}{l} \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \end{array} \right)$

no source of charge ($\vec{J}$)

integral form! $\quad \int \vec{\nabla}\cdot\vec{J}\, d\tau = \oint \vec{J}\cdot d\vec{a} = 0$



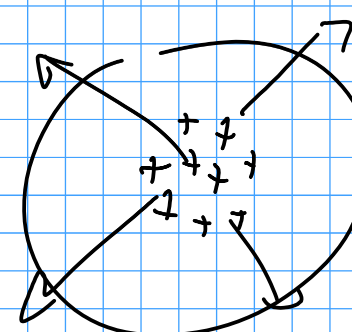
$$\int_{\text{left}} \vec{J}\cdot da + \int_{\text{right}} \vec{J}\cdot d\vec{a} = 0$$

dyn

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} \qquad \oint \vec{J}\cdot d\vec{a} = -\frac{\partial}{\partial t} Q_{encl}$$

$$\oint \vec{J}\cdot d\vec{a} = -\frac{\partial}{\partial t} Q_{encl}$$

out of surface.



Ex: 1-D motion of cars on a freeway

$$-\frac{\partial \rho}{\partial t} = \vec{\nabla}\cdot\vec{J} \xrightarrow{\text{1-D}} \boxed{\frac{\partial(\lambda v)}{\partial x} = -\frac{\partial}{\partial t}\lambda}$$

integral form multiply both sides by dx

$$\frac{\partial}{\partial x}\rho v + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho v$$

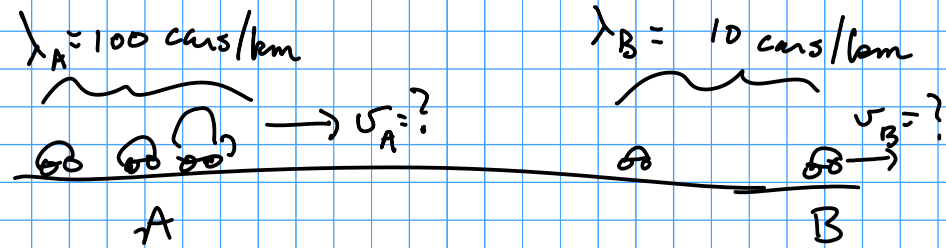
$$\int \frac{\partial}{\partial x}(\lambda v)\,dx = -\frac{\partial}{\partial t}\underbrace{\int \lambda\,dx}$$

$$\parallel$$

$$(\lambda v)_f - (\lambda v)_i = -\frac{\partial Q_{encl}}{\partial t}$$

Ex:

$$\lambda\left(\frac{C}{m}\right) \text{ or } \left(\frac{\text{Cars}}{\text{km}}\right)$$



steady state

$$(\lambda v)_B - (\lambda v)_A = 0$$

$$\lambda_B v_B = 10\,\frac{\text{cars}}{\text{km}}\,v_B$$

$$\lambda_A v_A = 100\,\frac{\text{cars}}{\text{km}}\,v_A$$

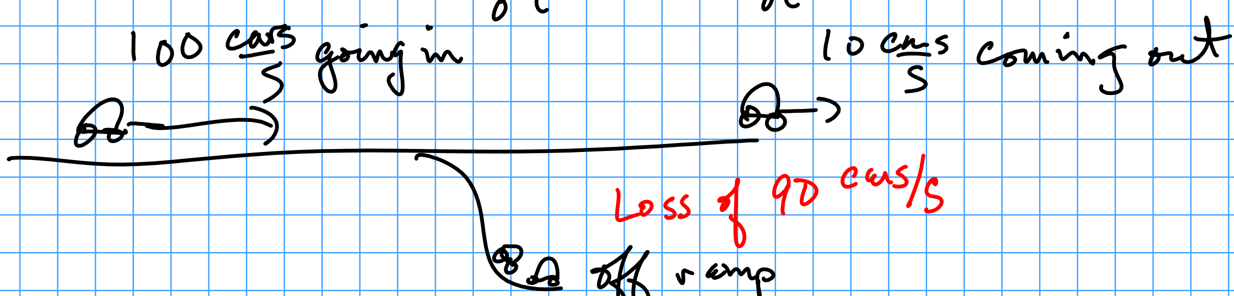
$$\Downarrow$$

$$\boxed{10\,v_B = 100\,v_a}$$

non-steady state: Let cars move at $1\,\frac{\text{km}}{\text{s}}$ at both A & B

$$\lambda_a v_a - \lambda_B v_B = (100-10)\,1\frac{\text{km}}{\text{s}} = 90\,\frac{\text{cars}}{\text{km}}\,\frac{\text{km}}{\text{s}} = 90\,\frac{\text{cars}}{\text{s}}$$

$$= -\frac{\partial}{\partial t}\int \lambda\,dx = -\frac{\partial}{\partial t}\left[\text{cars between A \& B}\right]$$



Cons in QM.

$$\rho \rightarrow \psi^* \psi \quad \frac{\text{prob}}{\text{vol}}$$

$$\vec{J} = \frac{\hbar}{m}\frac{1}{2i}\left(\psi^* \vec{\nabla}\psi - \psi \vec{\nabla}\psi^*\right) \leftarrow$$

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\psi^*\psi)}$$

Conservation prob

plane waves $\quad \psi = \psi_0 e^{i(kx-\omega t)} \quad ; \nabla\psi = \psi_0 ik e^{i(kx-\omega t)}$

$$J = \frac{\hbar}{m}\frac{1}{2i}\left(\psi^*\psi ik + \psi^*\psi ik\right) = \frac{\hbar}{m}k\psi^*\psi = \frac{\hbar}{m}\frac{m}{\hbar}v_g\psi^*\psi$$

$$p = \hbar k = m v_g \qquad k = \frac{m v_g}{\hbar}$$

$$J = \psi^*\psi v_g = \rho v_g \qquad \vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} \quad \rho = \psi^*\psi$$

$$J = \rho v_g$$