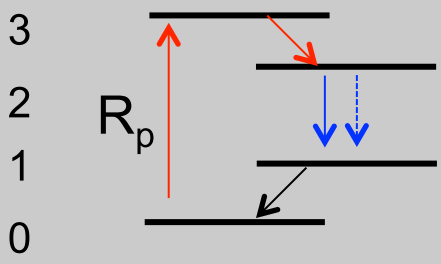
Modeling laser amplification and oscillation

- Energy input and output:
  - Input pump energy
  - Output power
  - Waste heat
- Internal *energy* balance:
  - Stored energy in inversion density
  - Circulating beam power
- Internal *number* balance:
  - Number of atoms inverted
  - Number of photons in cavity

Equations for lasing dynamics

- Consider a 4-level system:



Assume: τ_{32} and $\tau_{10} << \tau_{21}$ and $W_{21}N_2$

- Example: look at population dynamics for level 2

$$\frac{dN_2}{dt} = R_P - W\,N_2 - N_2/\tau_{21}$$

$R_p =$ Pump rate/volume

  – W is the stimulated emission rate

$$W = \rho\,B_{21} = \frac{2I}{c}\frac{c\sigma_{21}}{h\nu_{21}} = \frac{2\sigma_{21}}{h\nu_{21}}I$$

Factor of 2 b/c of both directions

- First investigate gain dynamics
  – Account for population changes during absorption and gain

Connect intensity changes to atomic rates

- In a volume V, absorbed power is $\frac{dP_a}{dV} = W_{12} N_1 h\nu$
- For a beam with area A, $\frac{dP_a}{dV} = \frac{1}{A}\frac{dP}{dz} = -\frac{dI}{dz}$
- Intensity and energy density are related: $\rho c = I$

$$\frac{dP_a}{dV} = -\frac{dI}{dz} = B_{12}\rho N_1 h\nu \qquad \frac{dI}{dz} = -I N_1 \frac{B_{12} h\nu}{c} = -I N_1 \sigma_{12}$$

$$\boxed{\sigma_{12} = \frac{B_{12} h\nu}{c}}$$

Note that the mean free path of photons in the medium is 1/α

Optical absorption

- With population in both levels 1 and 2,

$$\frac{dI}{dz} = I\left(N_2 B_{21} - N_1 B_{12}\right)\frac{h\nu_{21}}{c} \qquad B_{12}\frac{g_1}{g_2} = B_{21}$$

$$\frac{dI}{dz} = I\left(N_2 - N_1\frac{g_2}{g_1}\right)\frac{B_{21}h\nu_{21}}{c} = -I\,\Delta N\sigma_{21}$$

Inversion density

absorption cross-section

$$\boxed{I(z) = I_0 e^{-\alpha z}}$$

α: absorption coefficient = ΔN σ_{21}

For an amplifier of length L,

$$\boxed{I(L) = I_0 e^{-\alpha L} = I_0 T_0}$$

T_0: transmission

Optical absorption and gain

- With population in both levels 1 and 2,

$$\frac{dI}{dz} = I\left(N_2 B_{21} - N_1 B_{12}\right)\frac{h\nu_{21}}{c} \qquad B_{12}\frac{g_1}{g_2} = B_{21}$$

$$\frac{dI}{dz} = I\left(N_2 - N_1\frac{g_2}{g_1}\right)\frac{B_{21}h\nu_{21}}{c} = I\, N_{inv}\sigma_{21}$$

Inversion density

Gain cross-section

$$\boxed{I(z) = I_0 e^{gz}}$$

g: gain coefficient = $N_{inv}\ \sigma_{21}$
(opposite sign from absorption coefficient)

For an amplifier of length L,

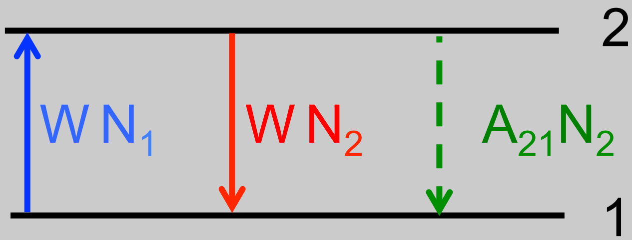
$$\boxed{I(L) = I_0 e^{gL} = I_0\, G_0}$$

G_0: small signal single-pass gain

Population dynamics of absorption

- Closed 2 level system, assume $g_1=g_2$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt}$$



$$\frac{dN_2}{dt} = W\,N_1 - W\,N_2 - A_{21}N_2$$

- Since system is closed, reduce to one equation for population difference: $\Delta N = N_1 - N_2$

$$N_t = N_1 + N_2$$

$$N_t = \Delta N + 2N_2$$

$$\rightarrow N_2 = \tfrac{1}{2}\left(N_t - \Delta N\right)$$

$$\frac{dN_1}{dt} - \frac{dN_2}{dt} = \frac{d}{dt}\Delta N = -2\frac{dN_2}{dt}$$

$$\frac{d}{dt}\Delta N = -2\left(W\,\Delta N - A_{21}N_2\right)$$

$$\frac{d}{dt}\Delta N = -2W\,\Delta N + A_{21}\left(N_t - \Delta N\right) = -\Delta N\left(A_{21} + 2W\right) + A_{21}N_t$$

  - Steady state: $\Delta N = \dfrac{N_t}{1 + 2W\tau_{21}}$ $\qquad A_{21} = 1/\tau_{21}$

Saturation of absorption

$$\Delta N = \frac{N_t}{1 + 2W\tau_{21}}$$

$$\Delta N = N_1 - N_2$$

- The key parameter in this situation is W τ_{21}

  $$W_{21} = \rho_\nu B_{21}$$

  - Low intensity, 2W τ_{21} << 1, $\Delta N \approx N_t$
  - High intensity, 2W τ_{21} >> 1, $\Delta N \approx 0$. Here $N_1 \approx N_2$
- Energy balance:

Input power
Absorbed by atoms → Radiated power (into 4π)
→ Stimulated emission (back into beam)

- Radiated power per unit volume:

$$\frac{dP}{dV} = h\nu_{21} W \Delta N(W) = h\nu_{21} \frac{N_t W}{1 + 2W\tau_{21}} \rightarrow h\nu_{21} \frac{N_t}{2\tau_{21}}$$

For W τ_{21} >> 1

Power radiated in high intensity limit: half of atoms are radiating

Saturation intensity

- Absorbed power per atom: $\sigma_{12} I$
- Absorption rate: $$W = \frac{\sigma_{12} I}{h\nu_{21}}$$
- In steady state: $$\frac{\Delta N}{N_t} = \frac{1}{1 + 2W\tau_{21}} = \frac{1}{1 + 2\frac{\sigma_{12} I}{h\nu_{21}}\tau_{21}} \equiv \frac{1}{1 + \frac{I}{I_{sat}}}$$
- Saturation intensity for absorption: $$I_{sat} = \frac{h\nu_{21}}{2\sigma_{12}\tau_{21}}$$
  - 2: transition affects both levels at once
  - At I = I_{sat}, stimulated and spontaneous emission rates are equal.
- Intensity-dependent absorption coefficient:

$$\alpha(I) = \frac{\alpha_0}{1 + I/I_{sat}}$$

At high intensity, material absorbs *less*.

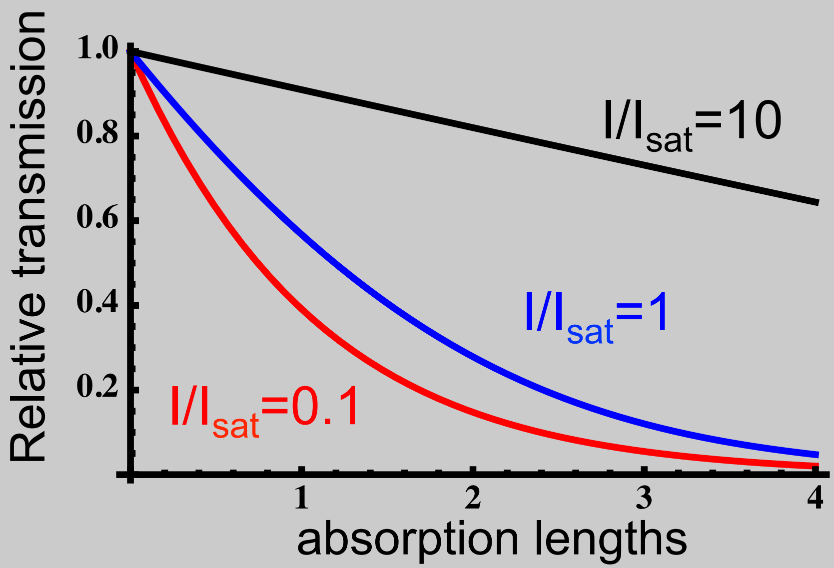
Saturable absorbers are used for pulsed lasers: Q-switching and mode-locking

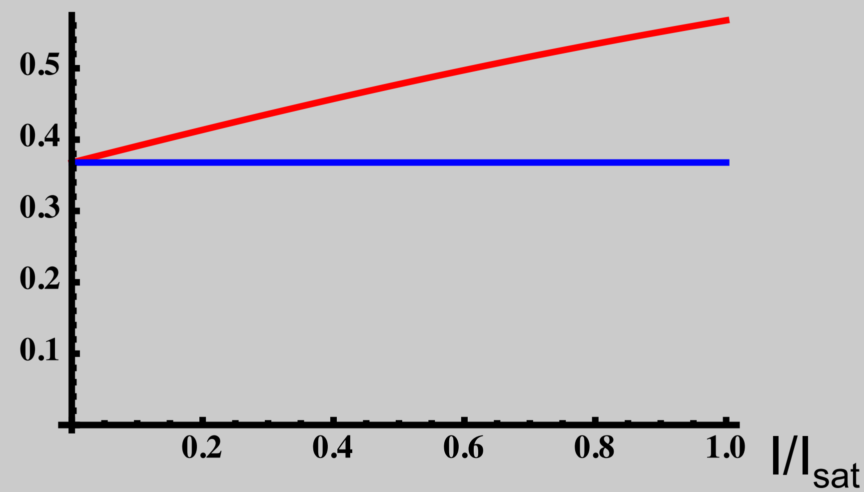
Saturated CW propagation through absorbing medium

- For a given thickness for an absorbing medium, the transmission will increase with intensity

$$\alpha(I) = \frac{\alpha_0}{1 + I/I_{sat}} \qquad \frac{dI}{dz} = -\alpha(I)I = -\frac{\alpha_0}{1+I/I_{sat}} I$$

$$\int_{I_0}^{I} \left(\frac{1}{I} + \frac{1}{I_{sat}} \right) dI = -\int_0^L \alpha_0 \, dz \rightarrow \ln\left[\frac{I(z)}{I(0)} \right] + \frac{I(z) - I(0)}{I_{sat}} = -\alpha_0 z$$





Pulsed input: saturation fluence

$$\Delta N = N_1 - N_2$$

Γ_{sat} = saturation fluence

- Rewrite equation using intensity:

$$\frac{d}{dt}\Delta N = -\Delta N\left(A_{21} + \frac{2\sigma}{h\nu_{21}} I(t)\right) + A_{21}N_t \equiv -\Delta N\left(A_{21} + \frac{I(t)}{\Gamma_{sat}}\right) + A_{21}N_t$$

- Scaling of equation
  - Two timescales: τ_p and τ_{21}, but pay attention to weighting

$$\frac{d}{dt}\Delta N = -\Delta N\left(\frac{1}{\tau_{21}} + \frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}\right) + \frac{1}{\tau_{21}}N_t = \frac{2N_2}{\tau_{21}} - \frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}\Delta N$$

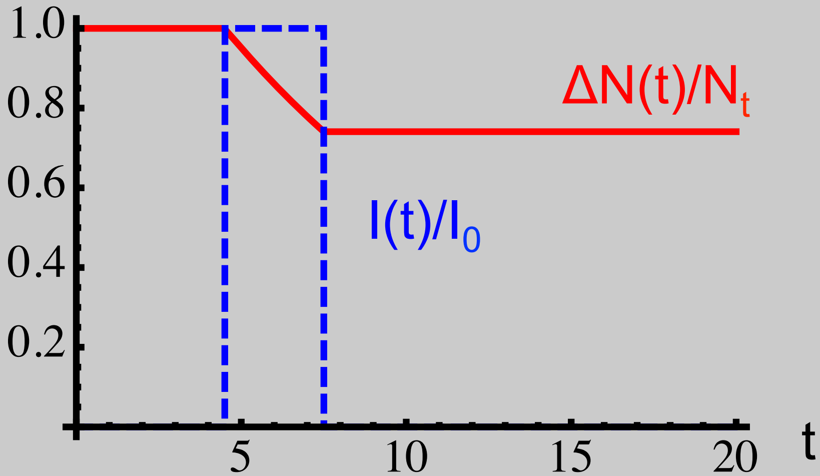
- Short pulse input: ignore fluorescence, assume $\Gamma_{in}/\Gamma_{sat}, |\Delta N| \approx 1$

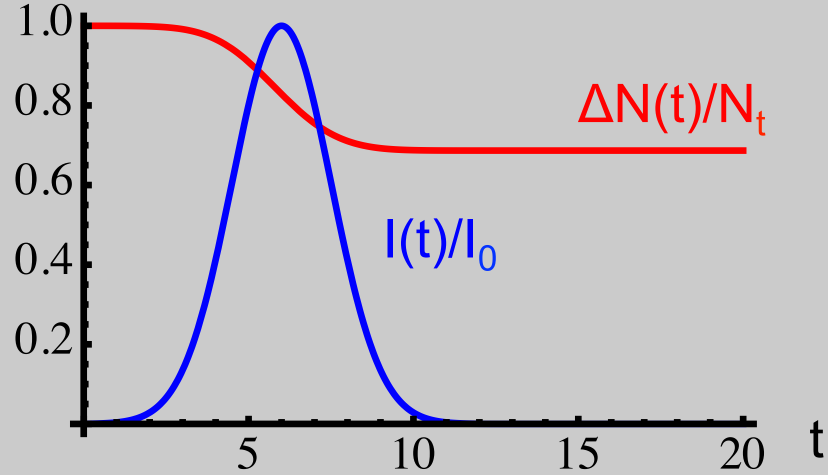
$$\frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}|\Delta N| \gg \frac{2N_2}{\tau_{21}} \quad \rightarrow \frac{d}{dt}\Delta N \approx -\frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}\Delta N \quad \rightarrow \ln\left[\frac{\Delta N(t)}{\Delta N(0)}\right] \approx -\frac{1}{\Gamma_{sat}}\int_0^t I(t')dt'$$

$$\rightarrow \Delta N(t) = N_1(t) - N_2(t) \approx N_t \exp\left[-\frac{1}{\Gamma_{sat}}\int_0^t I(t')dt'\right] = N_t \exp\left[-\frac{\Gamma_{in}}{\Gamma_{sat}}\right]$$

Short pulse limit

- For short pulse input: $\tau_p \ll \tau_{21}$, so ignore fluorescence
  - Medium just integrates energy of pulse.
  - Example: Ti:sapphire: τ_{21}=3.2μs, τ_p=10ns or 200ns for Q-switched Nd:YAG lasers pumped with flashlamps or CW arc lamps
- Square input pulse   Gaussian input pulse
  - $\tau = 3$, $I_0/I_{sat} = 0.1$, (no fluorescence)





- Shape of *transmitted* pulse is affected

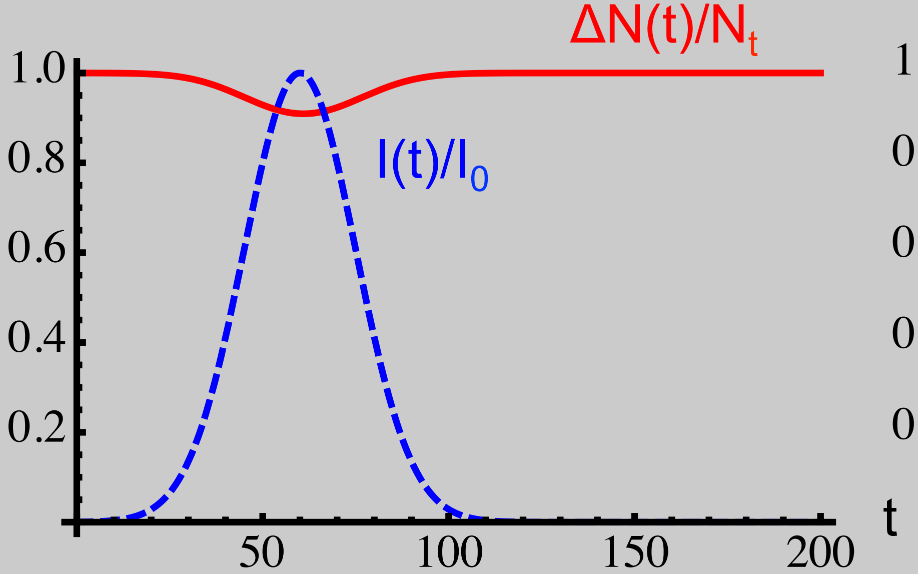
Long pulse limit

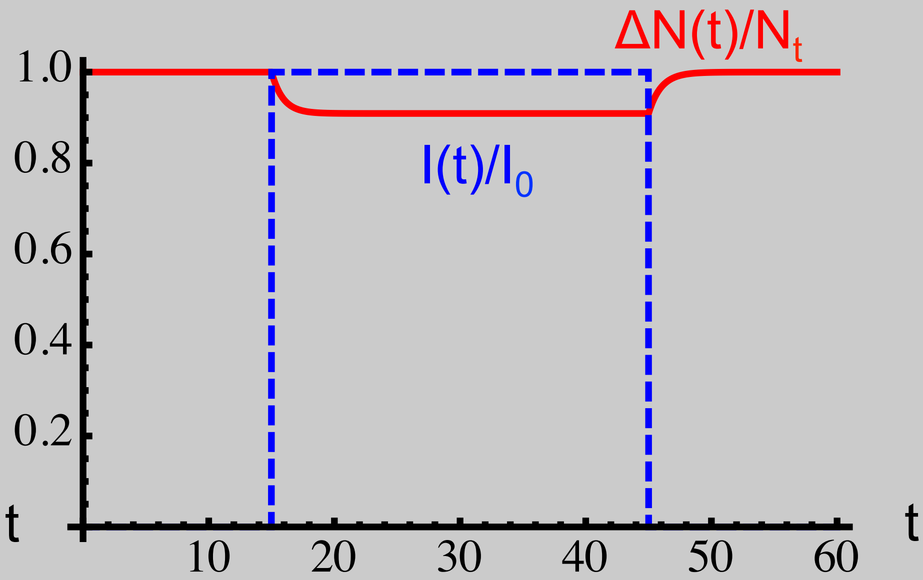
- For *long* pulse input: $\tau_p \gg \tau_{21}$, and peak $I \ll I_{sat}$, $\Delta N(t)$ follows $I(t)$

$$\rightarrow \frac{d}{dt}\Delta N \ll A_{21} N_t$$

Quasi-static, quasi-CW limit
N_t adiabatically follows I(t)

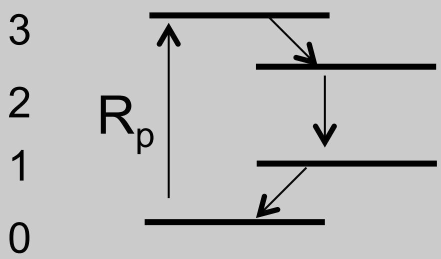
$$\frac{\Delta N}{N_t} = \frac{1}{1 + I(t)/I_{sat}}$$





Gain saturation

- Consider a 4-level system:



Assume: τ_{32} and τ_{10} << τ_{21} and $W_{21}N_2$

- Look at level 2 only:

$$\frac{dN_2}{dt} = R_P - W\,N_2 - N_2/\tau_{21}$$

Low intensity: $N_2 = R_P\tau_{12}$

τ_{12} is called "storage time"

- Steady state: $$N_2 = \frac{R_P\tau_{21}}{1+W\tau_{21}} = \frac{R_P\tau_{21}}{1+\frac{\sigma_{21}\tau_{21}}{h\nu_{21}}I} = \frac{R_P\tau_{21}}{1+\frac{I}{I_{sat}}}$$

- Saturation intensity for *gain:*
  - *No factor of 2*

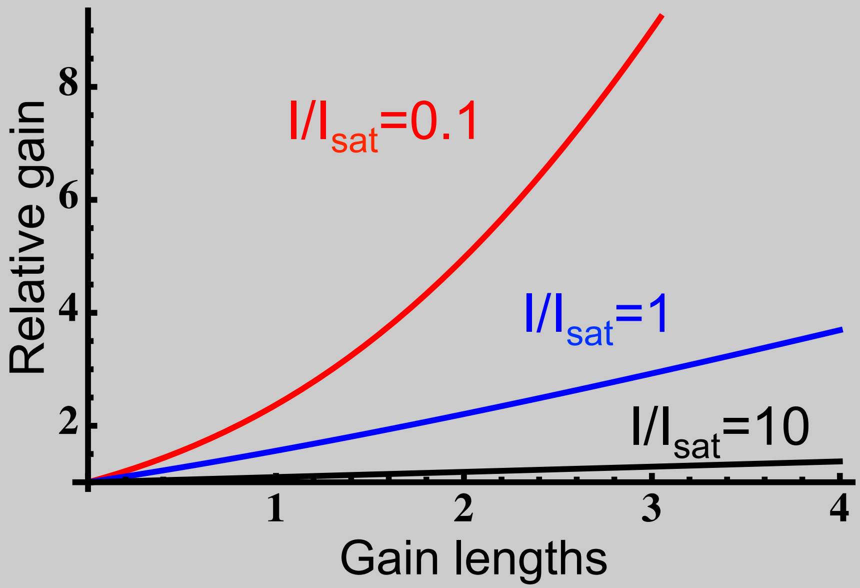
$$I_{sat} = \frac{h\nu_{21}}{\sigma_{21}\tau_{21}} = \frac{\Gamma_{sat}}{\tau_{21}}$$

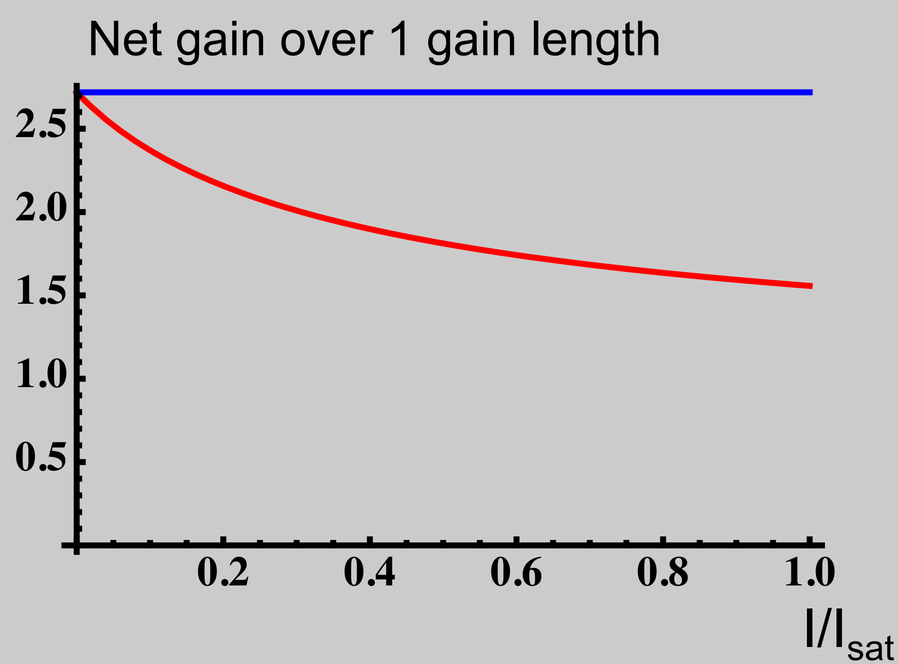
$$g(I) = \frac{g_0}{1+I/I_{sat}}$$

Beam growth during amplification

- Calculation just as with absorption

$$\int_{I_0}^{I}\left(\frac{1}{I}+\frac{1}{I_{sat}}\right)dI=+\int_0^L g_0\,dz \rightarrow \ln\left[\frac{I(z)}{I(0)}\right]+\frac{I(z)-I(0)}{I_{sat}}=+g_0 z$$





- **Even though saturated gain is low, it is efficient at extracting stored energy**

Spatial dependence of gain

- Gain follows distribution of pump intensity
- Spatial variation of gain affects beam profile
- Examples:
  - longitudinal pumping with Gaussian beam leads to gain narrowing of spatial profile. More gain in center, less at edges
  - Saturated absorption by a Gaussian beam: saturation in center suppresses intensity there. Leads to widening of output beam.

Saturated gain for pulse amplification

- Starting with a square pulse in time entering a gain medium, integrating the time-dependent gain including saturation effects leads to Frantz-Nodvick equation:

$$G = \frac{\Gamma_{sat}}{\Gamma_{seed}} \ln[1 + (e^{\Gamma_{seed}/\Gamma_{sat}} - 1)e^{\Gamma_{Pump}/\Gamma_{Sat}}]$$

  – In the limit of small input fluence,

$$G \approx \frac{\Gamma_{sat}}{\Gamma_{seed}} \ln[1 + \frac{\Gamma_{seed}}{\Gamma_{sat}} e^{\Gamma_{Pump}/\Gamma_{Sat}}] \approx e^{\Gamma_{Pump}/\Gamma_{Sat}} = G_0$$

- This expression is often used in modeling amplifiers

Pulse amplification: saturated gain algorithm

Frantz-Nodvick Equation:

$$G = \frac{\Gamma_{sat}}{\Gamma_{seed}} \ln[1 + (e^{\Gamma_{seed}/\Gamma_{sat}} - 1)e^{\Gamma_{Pump}/\Gamma_{Sat}}]$$

No Spatial Dependence:

$$F_{Pump} = \frac{E_{Abs}}{\pi w_P^2}$$

$$F_{Seed} = \frac{E_{Seed}}{\pi w_S^2}$$

→ **Frantz-Nodvick** → Gain

Assumptions:

- Thermal Equilibrium within Stark Manifolds
- Square Temporal Profile of Seed

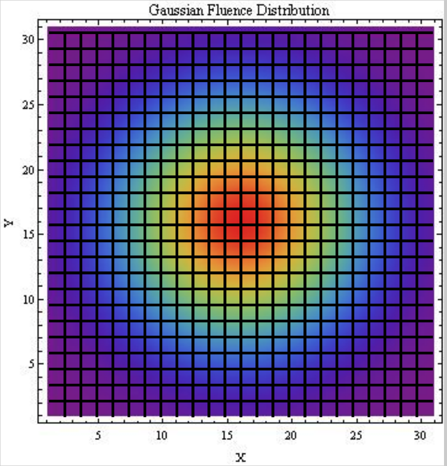
Transverse dependence: super-Gaussian

$$\Gamma(x,y) = \Gamma_0 e^{\frac{-[(\frac{x}{w_x})^{nx} + (\frac{y}{w_y})^{ny}]}{2}}$$

where:- ***nx,ny*** = 2 (Gaussian),
Even > 2 (Super-Gaussian)

-F_0 is defined via the Total Energy and integration of the distribution

(Δx, Δy) →



Define Spatial Arrays → **Define Gain Array** *(Frantz-Nodvick for each element in spatial array)* → $F'_{Seed} = G \bullet F_{Seed}$ → **Compute Post-Gain Seed Energy** (Integrate Spatial Array) →

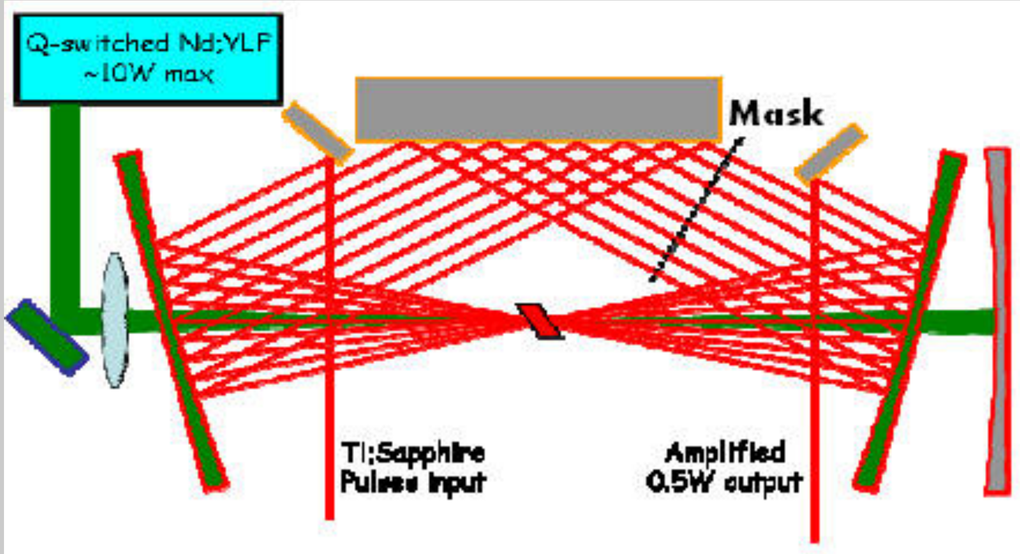
$$Gain = \frac{E'_{Seed}}{E_{Seed}}$$

Example: Ti:sapphire multipass amp

- Seed pulse from pulsed laser oscillator: 1nJ (800nm)
- Amplify to 1mJ, use 7mJ of pump energy (532nm)
- Multipass designs: spatially separate beams

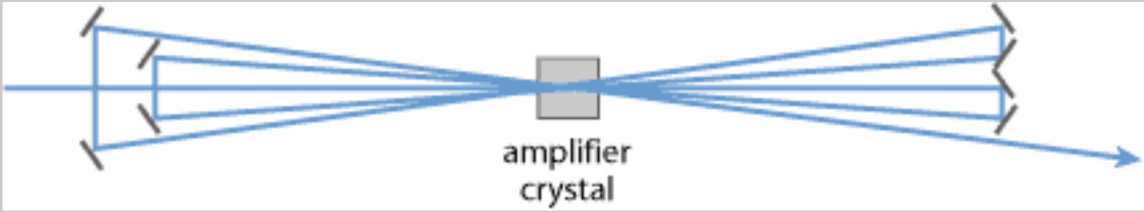
Three-mirror ring preamp:

- Up to 12 passes
- Focused beam in crystal
- 2 mirror alignment



Bowtie power amp:

- Collimated beam
- 8 mirrors



Multipass design

- Assume uniform pumping with round beams
- Calculate stored fluence and small signal gain
- Use saturated gain expression to calculate new energy after 1st pass
- Subtract extracted energy from stored energy (over seed spot area)
- Repeat for N passes

Conditions: 1nJ seed, 7mJ pump energy, 95% absorption, 10% loss/pass

Stored energy:

$$E_{stor} = E_{pump}\eta_{abs}\frac{h\nu_{seed}}{h\nu_{pump}} = 4.4mJ$$

Small signal gain estimate:

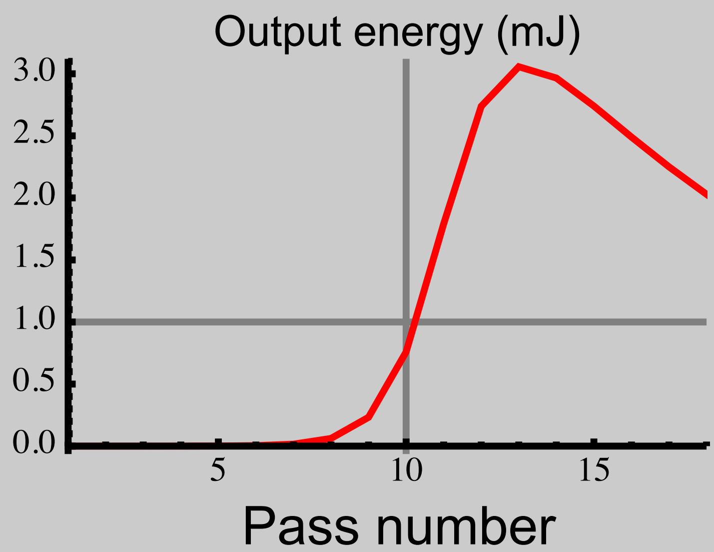
$$G_0 = \left(\frac{E_{\text{target}}}{E_{\text{seed}}}\right)^{1/N}\frac{1}{1-L} = 4.42$$

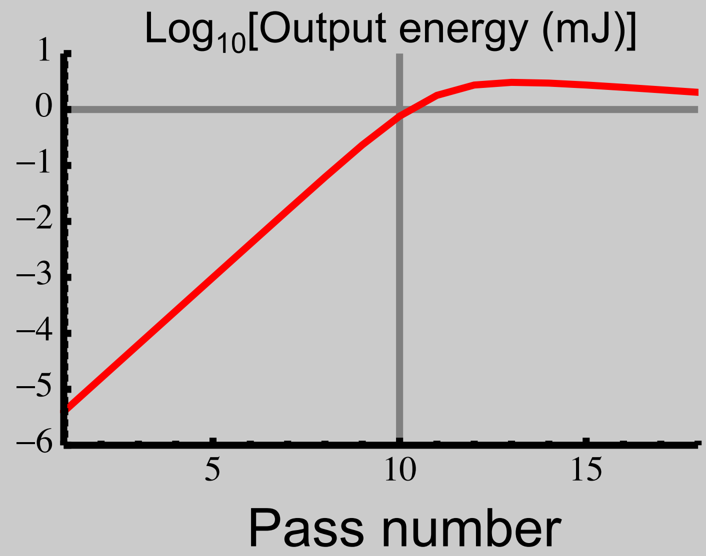
Estimated spot size:

$$A_{pump} = \frac{E_{stor}}{\Gamma_{sat}\ln\left[G_0\right]}, \quad w_p = 300\mu m$$

Multipass: Simple calculated results

- Small signal gain estimate works as long as stored energy is not depleted

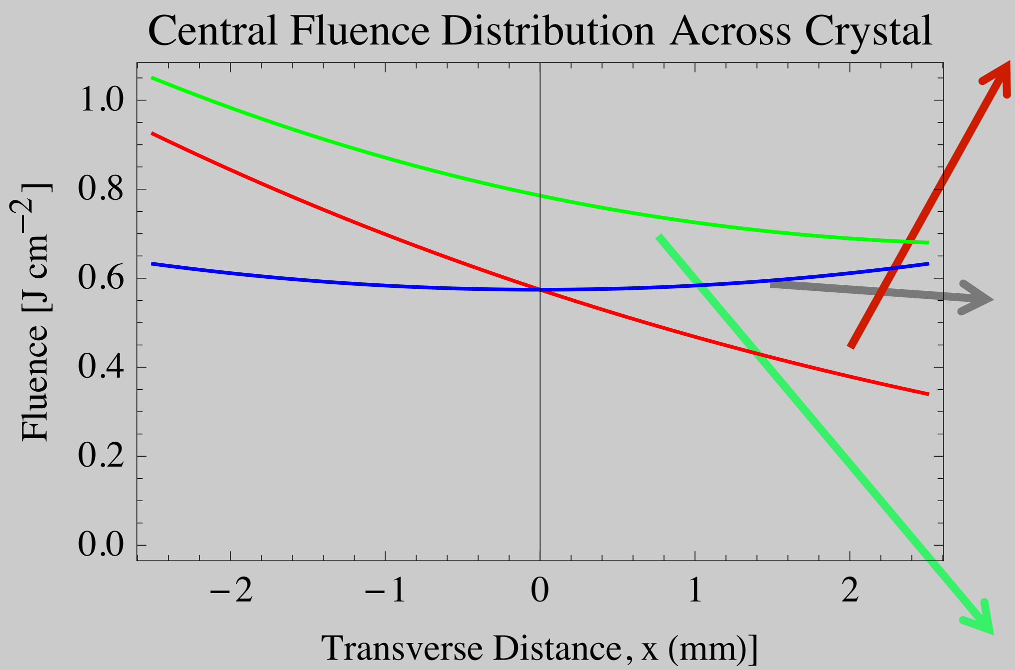


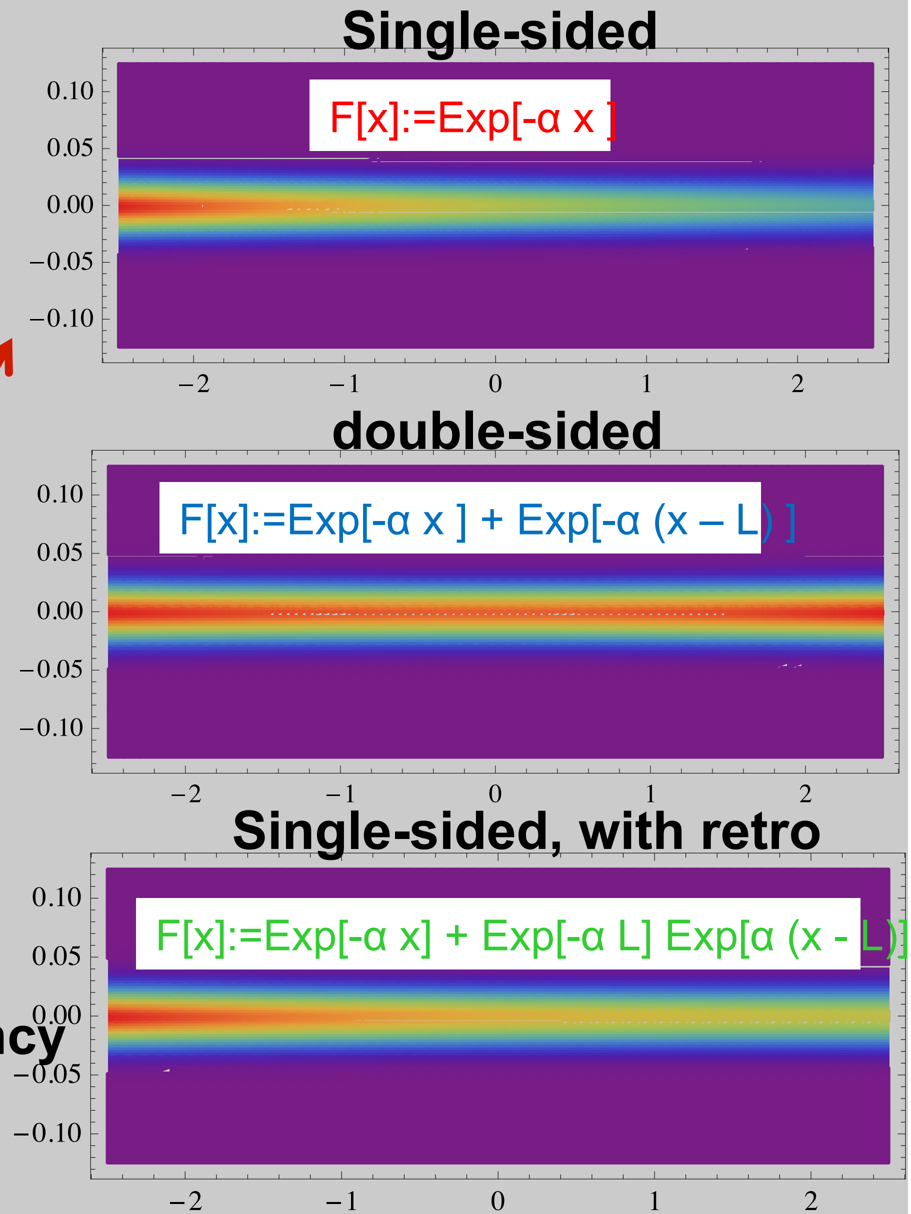


  - Smaller seed size to ensure full overlap with pump
  - Avoid damage thresholds for pump and seed
  - Saturate at desired energy to reduce noise
  - Account for size change in Brewster cut crystal

Transverse diode bar pumping

For good absorption, pump must have sufficient path length



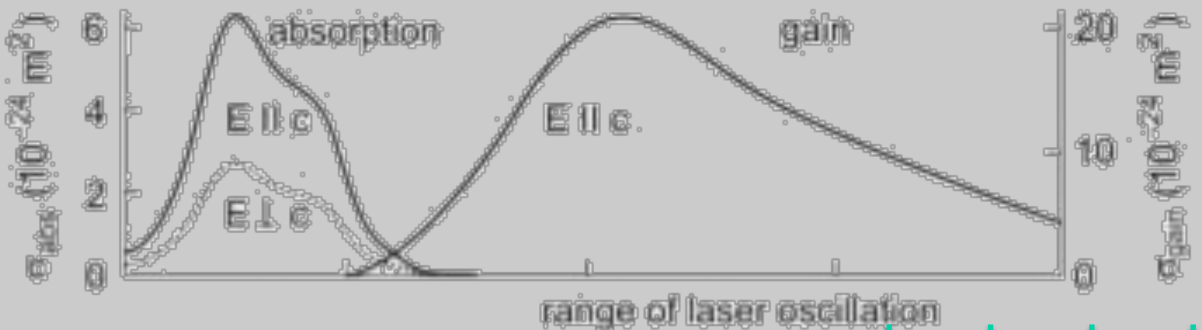


Using retro: better absorption efficiency

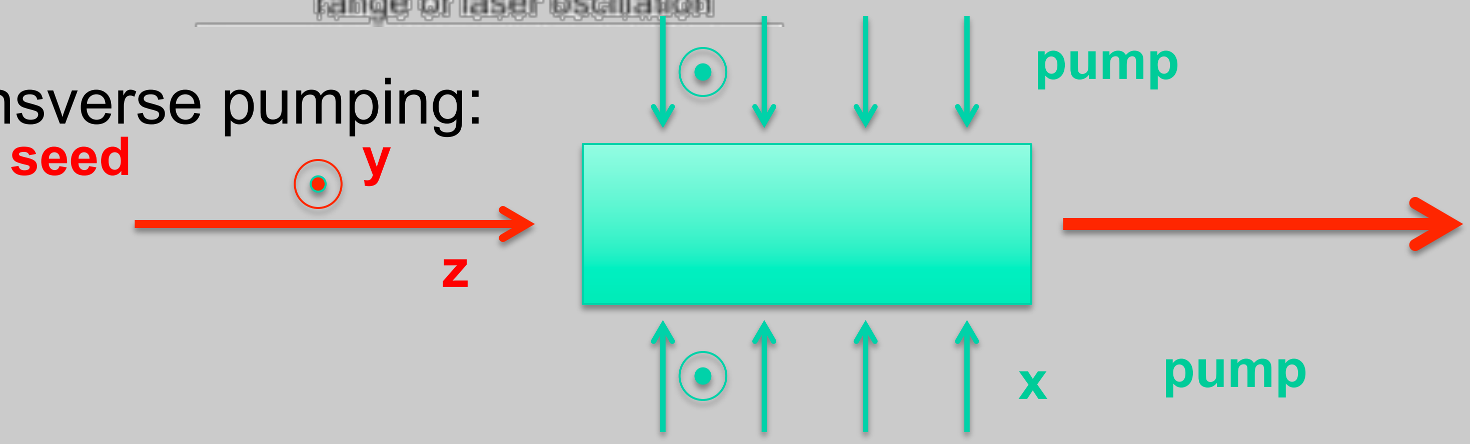
Double-sided: better uniformity

Polarization issues in pumping birefringent materials

- For Ti:sapphire, both polarizations contribute to seed gain along c-axis
- Much higher pump absorption for E along c-axis
  - α across c-axis is about 40% lower than along c-axis



- Ex: transverse pumping:



Frequency dependence: account for lineshapes

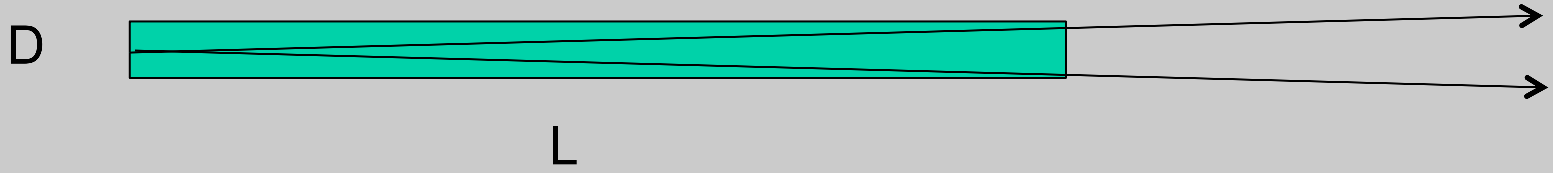
- Absorption and gain coefficients and saturation intensity both depends on frequency

$$\alpha(I,\nu) = \frac{\alpha_0(\nu-\nu_0)}{1+\frac{I(\nu)}{I_{sat}(\nu-\nu_0)}}$$

- For broadband input, saturation changes shape of transmitted spectrum
  - Absorption: power broadening
  - Gain: spectral gain narrowing

Amplified Spontaneous Emission (ASE)

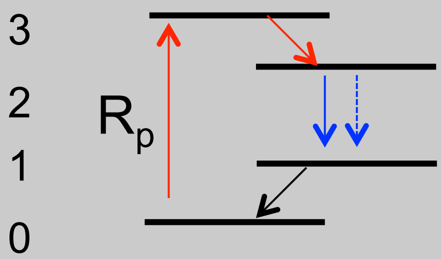
- Spontaneous emission is emitted into 4π steradians, but is amplified on the way out if there is gain.



  - ASE can be considered to be a noise source
  - ASE is more directional than fluorescence, but not as directional as a coherent laser beam
  - Some high-gain lasers are essentially ASE sources (no mirrors)
- Implications for amplifier design
  - ASE can deplete stored energy before pulse extraction
  - Use timing and good seed energy to extract energy from medium before ASE
  - Ensure that transverse gain is smaller than longitudinal to avoid parasitic depletion.

Equations for lasing dynamics

- Consider a 4-level system:



Assume: τ_{32} and τ_{10} << τ_{21} and $W_{21}N_2$

- Example: look at population dynamics for level 2

$$\frac{dN_2}{dt} = R_P - W N_2 - N_2 / \tau_{21}$$

$R_p =$ Pump rate/volume

  – W is the stimulated emission rate

$$W = \rho B_{21} = \frac{2I}{c}\frac{c\sigma_{21}}{h\nu_{21}} = \frac{2\sigma_{21}}{h\nu_{21}} I$$

Factor of 2 b/c of both directions

- We will want to keep track of two variables:
  – N_2: population inversion density (#/vol)
  – ϕ: total number of photons in cavity mode

Rate equation for population inversion

- Represent stimulated emission in terms of ϕ
  - Let $W \equiv B\phi$ where B is the stimulated emission rate per photon

$$\frac{dN_2}{dt} = R_P - W N_2 - N_2/\tau_{21} \rightarrow \boxed{\frac{dN_2}{dt} = R_P - B\phi N_2 - N_2/\tau_{21}}$$

  - Calculate B in terms of B_{21} and σ_{21}

$$W = \rho B_{21} = B\phi \qquad \rightarrow W = \frac{\phi h\nu_{21}}{V}\frac{c\sigma_{21}}{h\nu_{21}} = B\phi \rightarrow B = \frac{c\sigma_{21}}{V} = B_{21}\frac{h\nu_{21}}{V}$$

Energy density $\rho = \frac{\phi h\nu_{21}}{V}$

$B_{21} = \frac{c\sigma_{21}}{h\nu_{21}}$ Einstein B

$V = A_b L$ Cavity mode volume

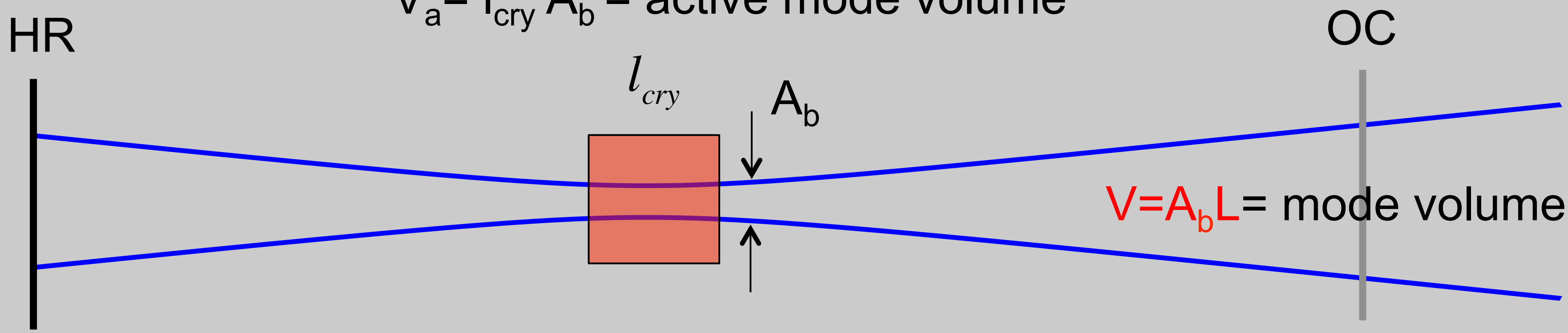
$$B = \frac{c}{A_b L}\sigma_{21} \rightarrow \frac{2\sigma_{21}}{A_b T_{RT}}$$

Round trip time

$$T_{RT} = \frac{2L}{c}$$

Model laser cavity

$V_a = l_{cry} A_b$ = active mode volume



$$L = L_{cav} + l_{cry}(n-1)$$

$$T_{RT} = \frac{2L}{c}$$ Round-trip time

$$\phi = \frac{\rho V}{h\nu_{21}}$$ Total number of photons in cavity

$$\rho = \frac{2I}{c}$$ Energy density. 2 I = average intensity (both directions), neglecting standing wave interference

$$\phi = \frac{2I}{c}\frac{L A_b}{h\nu_{21}} = I\frac{A_b T_{RT}}{h\nu_{21}}$$

Photon gain rate

- Since *B* is the stimulated emission rate per photon

$$\frac{d\phi}{dt} = BV_a N_2 \phi$$

$$V_a N_2 = \text{number of atoms capable of contributing to beam}$$

- Alternative: each photon added to beam comes from stimulated emission transition from N_2

$$\frac{d\phi}{dt} = V_a \frac{dN_2}{dt} = V_a B_{21} \rho N_2$$

Energy density

$$\rho = \frac{\phi}{V} h\nu_{21}$$

From previous slide:

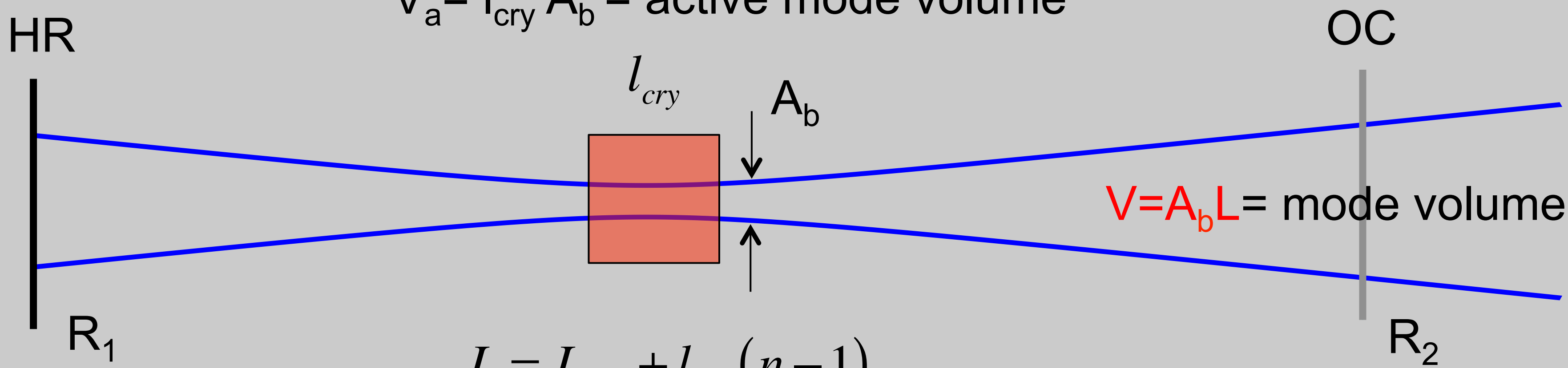
$$B = B_{21} \frac{h\nu_{21}}{V} \rightarrow B_{21} = B \frac{V}{h\nu_{21}}$$

$$\rightarrow \frac{d\phi}{dt} = V_a B \frac{V}{h\nu_{21}} \frac{\phi}{V} h\nu_{21} N_2$$

$$\rightarrow \frac{d\phi}{dt} = BV_a N_2 \phi$$

Model laser cavity

$V_a = l_{cry} A_b$ = active mode volume



$$L = L_{cav} + l_{cry}(n-1)$$

$\mathcal{L}_i$ Internal passive losses

$$T_{RT} = \frac{2L}{c}$$ Round-trip time

$$\phi = \frac{\rho V}{h\nu_{21}}$$ Total number of photons in cavity

$$\phi = \frac{2I}{c}\frac{L A_b}{h\nu_{21}} = I\frac{A_b T_{RT}}{h\nu_{21}}$$

Passive cavity loss rate

- Light losses in cavity from
  - mirrors (output coupling and leakage)
  - internal losses (reflections, absorption, scatter, misalignment)
- Set up differential equation
  - Assume losses are small, can be approximated as evenly distributed
  - After *m* passes through gain medium: $t_m = mT_{RT}/2$

$$\phi(t_m) = \left[R_1 R_2 (1-\mathcal{L}_i)^2\right]^{m/2} \phi_0$$

$$\rightarrow \phi(t_m) = e^{-m\gamma}\phi_0$$

Define γ so that

$$e^{-\gamma} \equiv \sqrt{R_1 R_2}(1-\mathcal{L}_i)$$

$$\gamma = -\ln\left[\sqrt{R_1 R_2}(1-\mathcal{L}_i)\right]$$

$$\frac{\phi(t_m)-\phi(t_{m-1})}{T_{RT}/2} = \frac{e^{-\gamma}\phi_m - \phi_{m-1}}{T_{RT}/2} \rightarrow \frac{d\phi}{dt} = -\left(\frac{1-e^{-\gamma}}{T_{RT}/2}\right)\phi \approx -\frac{2\gamma}{T_{RT}}\phi$$

$$\frac{d\phi}{dt} = -\frac{\phi}{\tau_c}$$

$$\tau_c = T_{RT}/2\gamma = L/\gamma c$$

Photon cavity lifetime

Separate loss terms:

$$\gamma = \gamma_i + \tfrac{1}{2}\gamma_1 + \tfrac{1}{2}\gamma_2$$

Equations for laser dynamics

- Combine gain and loss terms for photon number

$$\frac{d\phi}{dt} = V_a B N_2 \phi - \frac{\phi}{\tau_c}$$

$$\frac{dN_2}{dt} = R_P - B\phi N_2 - N_2 / \tau_{21}$$

Can add a photon for vacuum contribution

$$\frac{d\phi}{dt} = V_a B N_2 \left(\phi + 1\right) - \frac{\phi}{\tau_c}$$

this allows build-up to get started

- **Output power:** from mirror M_2

$$\text{OC transmission} = 1 - R_2$$

$$\text{output power} = \left(1 - R_2\right) \cdot \text{intracavity power}$$

$$P_{out} = \gamma_2 \frac{h\nu_{21}}{T_{RT}} \phi = \gamma_2 \frac{c}{2L} h\nu_{21} \phi$$

Separate loss terms:

$$\gamma = \gamma_i + \tfrac{1}{2}\gamma_1 + \tfrac{1}{2}\gamma_2$$

Lasing threshold

- At threshold, gain balances loss

$$\frac{d\phi}{dt} = \left(V_a B N_2 - \frac{1}{\tau_c} \right) \phi \qquad V_a B N_2 \geq \frac{1}{\tau_c} \quad \text{for net gain}$$

  - *Critical inversion density* N_c

$$\tau_c = L / \gamma c$$

$$N_c = \frac{1}{V_a B \tau_c} = \frac{1}{V_a} \frac{V}{\sigma_{21} c} \frac{\gamma c}{L} = \frac{L}{l_{cry}} \frac{1}{\sigma_{21}} \frac{\gamma}{L}$$

$$B = \frac{\sigma_{21} c}{V}$$

$$\boxed{N_c = \frac{\gamma}{\sigma_{21} l_{cry}}}$$

  - At threshold, $\phi \approx 0$, $N_2 = N_c$

$$\frac{dN_2}{dt} = 0 = R_{cp} - N_c / \tau_{21}$$

  - *Critical pumping rate*:

$$\boxed{R_{cp} = \frac{N_c}{\tau_{21}} = \frac{\gamma}{\sigma_{21} l_{cry} \tau_{21}}}$$

Lasing above threshold

- pumping rate exceeds the critical value
- Steady state: time derivatives = 0
- Find steady-state values: N_0 and ϕ_0

$$\frac{d\phi}{dt} = V_a B N_2 \phi - \frac{\phi}{\tau_c}$$

$$\frac{dN_2}{dt} = R_P - B\phi N_2 - N_2/\tau_{21}$$

$$\frac{d\phi}{dt} = 0 = V_a B N_0 \phi_0 - \frac{\phi_0}{\tau_c} \rightarrow \left(V_a B N_0 - \frac{1}{\tau_c}\right)\phi_0 = 0$$

$$B = \frac{\sigma_{21} c}{V}$$

$$\therefore N_0 = \frac{1}{V_a B \tau_c} = N_{th}$$

Steady-state inversion density is clamped at N_{th}

$$\frac{dN_2}{dt} = 0 = R_P - B\phi_0 N_0 - N_0/\tau_{21} \rightarrow \phi_0 = \frac{R_P - N_0/\tau_{21}}{BN_0}$$

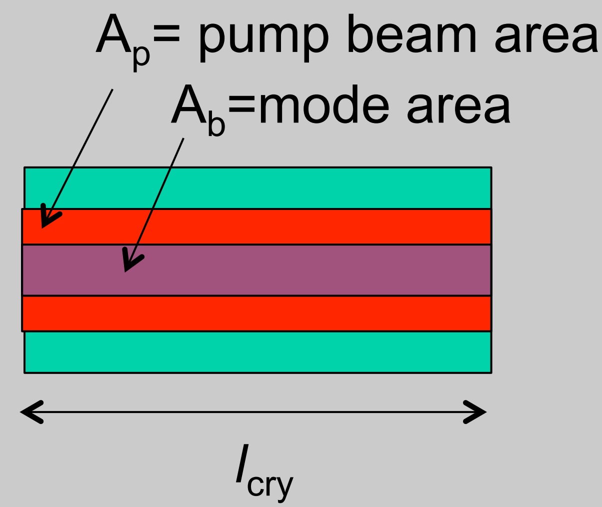
Steady-state photon number

$$\phi_0 = \tau_c\left(V_a R_P - \frac{V_a N_0}{\tau_{21}}\right) = \text{cavity storage time}(\text{atom pump rate - fluor loss rate})$$

$$= V_a \tau_c \left(R_P - \frac{N_0}{\tau_{21}}\right)$$

Modeling a longitudinally-pumped laser

- Longitudinal laser pumping is the easiest to calculate the inversion density
- Assume:
  - beam cross-section is flat-topped
  - $A_p \geq A_b$



$$R_p = \eta_{QE} \frac{P_{abs}}{h\nu_p} \frac{1}{A_p l_{cry}}$$

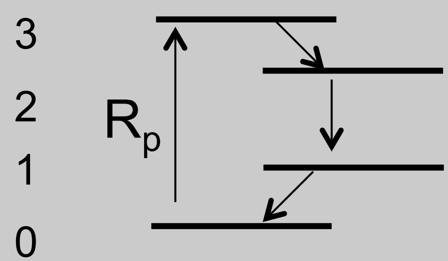
Volume of pump absorption

Photon arrival rate

P_{abs} = absorbed pump power = $P_{in}\eta_{abs}$, η_{abs} = (1-exp[-α_{03} l_{cry}])

also account for any pump delivery loss

η_{QE} = quantum efficiency for transfer from pump band to upper level



For Nd:YAG, η_{QE} = 0.95%

$$\rightarrow R_p = \eta_{QE}\eta_{abs} \frac{P_{in}}{h\nu_p} \frac{1}{A_p l_{cry}}$$

Pump threshold power

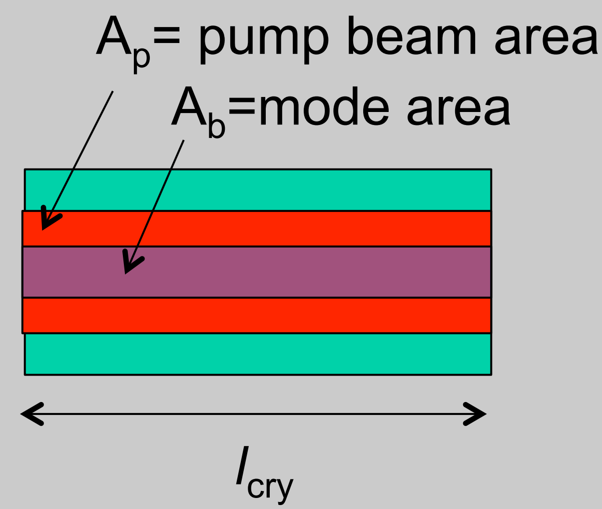
- Connect pump rate to incident pump power

$$R_p = \eta_{QE}\eta_{abs}\frac{P_{in}}{h\nu_p}\frac{1}{A_p l_{cry}}$$

- Critical pumping rate at threshold

$$R_{cp} = \frac{\gamma}{\sigma_{21} l_{cry}}\frac{1}{\tau_{21}} = \eta_{QE}\eta_{abs}\frac{P_{th}}{h\nu_p}\frac{1}{A_p l_{cry}}$$

$$\rightarrow P_{th} = \frac{\gamma}{\eta_{QE}\eta_{abs}\sigma_{21}}\frac{h\nu_p}{\tau_{21}}A_p = I_s A_p \frac{\gamma}{\eta_{QE}\eta_{abs}}\frac{h\nu_p}{h\nu_{21}}$$



$$I_s = \frac{h\nu_{21}}{\sigma_{21}\tau_{21}}$$

Note ratio of photon energies of pump and lasing photons

$$\eta_S = \frac{h\nu_{21}}{h\nu_p} = \text{Stokes ratio}$$

Output power

- The steady-state, circulating photon number is

$$\phi_0 = V_a \tau_c \left(R_P - \frac{N_0}{\tau_{21}} \right) = V_a N_0 \frac{\tau_c}{\tau_{21}} \left(\frac{R_P \tau_{21}}{N_0} - 1 \right)$$

$$R_{cp} = \frac{N_0}{\tau_{21}}$$

$$\phi_0 = \frac{A_b \gamma}{\sigma_{21}} \frac{\tau_c}{\tau_{21}} \left(\frac{P_P}{P_{th}} - 1 \right)$$

$$N_0 = \frac{\gamma}{\sigma_{21} l} = N_{th}$$

$$\frac{V_a}{l} = A_b \qquad \frac{R_p}{R_{cp}} = \frac{P_{in}}{P_{th}}$$

- Output power

$$P_{out} = \gamma_2 \frac{h\nu_{21}}{T_{RT}} \phi_0 = \gamma_2 \frac{c}{2L} h\nu_{21} \phi_0$$

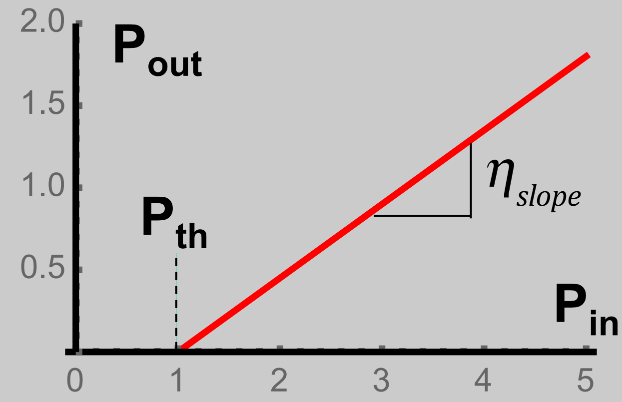
$$\rightarrow P_{out} = \gamma_2 \frac{c}{2L} h\nu_{21} \frac{A_b \gamma}{\sigma_{21}} \frac{\tau_c}{\tau_{21}} \left(\frac{P_P}{P_{th}} - 1 \right)$$

$$\tau_c = L/\gamma c$$

$$= \frac{\gamma_2}{2} \frac{c}{L} \frac{h\nu_{21}}{\sigma_{21}\tau_{21}} A_b \gamma \frac{L}{\gamma c} \left(\frac{P_p}{P_{th}} - 1 \right) = \frac{\gamma_2}{2} A_b I_{sat} \gamma \left(\frac{P_p}{P_{th}} - 1 \right)$$

Quantifying laser performance

- Measure output power vs input power
  - Get threshold power and slope efficiency



$$P_{th} = I_s A_p \frac{\gamma}{\eta_{QE}\eta_{abs}\eta_S}$$

$$\eta_s = \frac{I_s A_b}{P_{th}}\frac{\gamma_2}{2} = \frac{I_s A_b}{I_s A_p \frac{\gamma}{\eta_{QE}\eta_{abs}\eta_S}}\frac{\gamma_2}{2} = \frac{A_b}{A_p}\eta_{QE}\eta_{abs}\eta_S\frac{\gamma_2}{2\gamma}$$

$$\eta_b = \frac{A_b}{A_p} = \text{beam overlap efficiency} \qquad \eta_{OC} = \frac{\gamma_2}{2\gamma} = \text{output coupling efficienc}$$

$$\eta_s = \eta_b\eta_{QE}\eta_{abs}\eta_S\eta_{OC}$$

Our system design aims to optimize each of these efficiencies.

Extensions of model

- Assumptions so far:
  - spatially uniform pumping, uniform beam profile
  - Fast depletion from level 1 to ground state
- Other complications
  - Spatially-dependent pump distribution
  - Efficiency in producing pump energy (leads to “wall-plug efficiency)
  - 3-level and quasi- 3-level systems
  - Inhomogeneous broadening
  - Transients…