Homework 9 due 7 Nov. 2007 by 5pm
PH462 EM Waves and Optical Physics posted: 31 Oct. 2007

1) HM problem 9-21.

2) HM problem 9-22.

3) When the dipole radiates because it is driven by an external wave, the incident radiation is *scattered*. We can use this description of scattering when the scattering object is much less than the wavelength.

   a. A forced, damped oscillator has a position dependence:

   $$x(t) = \frac{-eE_0}{m_e} \frac{e^{-i\omega t}}{\left(\omega_0^2 - \omega^2\right) - i\nu\omega} ,$$

   where ω_0 is the resonance frequency, and ν is the radiative damping rate for a particular resonance (see class notes for the radiation damping of a freely oscillating charge). Calculate the total radiated power, using the Larmor equation (eq. 8-89 in HM). To calculate the acceleration, do the time derivatives, then take the real part, that is $\langle a^2 \rangle = \left\langle (\mathrm{Re}[\ddot{x}])^2 \right\rangle$.

   b. The scattering cross-section σ is calculated by dividing the radiated power by the incident intensity: $\sigma = P_{avg} / I_{inc}$. Show that the scattering cross-section is given by:

   $$\sigma = \frac{8\pi r_e^2}{3} \frac{\omega^4}{\left(\omega_0^2 - \omega^2\right)^2 + (\nu\omega)^2},$$ where the classical electron radius is $r_e = e^2 / m_e c^2$.

   c. Show that in the low frequency limit, i.e. $\omega << \omega_0$, $\nu << \omega_0$, $\sigma = \frac{8\pi r_e^2}{3}\left(\frac{\omega}{\omega_0}\right)^4$.

   In this case, we have Rayleigh scattering, with the characteristic ω^4 dependence. (One reason why the sky is blue.)

   d. Show that in the high frequency limit, i.e. $\omega >> \omega_0$, $\sigma = \frac{8\pi r_e^2}{3}$. This is the limit of Thomson scattering, the scattering from a free electron. When the frequency of the incident light is above all resonances (for example soft x-rays incident on atoms), the electrons behave as if they are free.

   e. Make a plot of $\mathrm{Log}_{10}[\sigma]$ vs ω/ω_0 for a range near $0 < \omega/\omega_0 < 3$, for two damping rates, $\nu = 0.1\ \omega_0$ and $\nu = 0.02\ \omega_0$.

4) Consider a collection of free electrons that have a thermal velocity distribution (Boltzmann). The Thomson-scattered light will be Doppler shifted ($\Delta\omega = \pm\omega_0 v / c$, where ω_0 is the angular frequency of the incident light).

   a. Show that the spectrum of the scattered light is Gaussian in profile.

   b. Derive an expression for the full-width at half maximum (FWHM) of the spectrum and the temperature of the plasma. What mean thermal energy (kT) in eV would correspond to a measured width of $\Delta\omega / \omega_0 = 5\mathrm{x}10^{-3}$?

5) HM problem 10.4.