9/20

Note Title 9/20/2006

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$x + 2y = 1$

$3x + 4y = 2$

$x = 1 - 2y$

$\Downarrow$

$3(1-2y) + 4y = 2$

$3 - 6y + 4y = 2$

$-2y = -1$

$y = 1/2$

$x = 1 - 1 = 0$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{array}{c|cc|c} ① & 1 & 2 & 1 \\ ② & 3 & 4 & 2 \end{array}$$

subtract $3 \times$ ① from ②

$$\begin{array}{ccc} 1 & 2 & 1 \\ 0 & -2 & -1 \end{array}$$

add ① + ②

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & -2 & -1 \end{array}$$

$$\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1/2 \end{array}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$$

$x = 0$

$y = 1/2$

$Ax = y$ $\qquad$ A^{-1}

$$\left.\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right.$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left.\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right.$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$x + 2y = 0$ $\qquad$ $x = -2y$

$3x + 4y = 0$

$3(-2y) + 4y = 0$

$-6y + 4y = 0$

$-2y = 0$

True $\Longleftrightarrow$ $y = 0$ $\quad$ $x = 0$

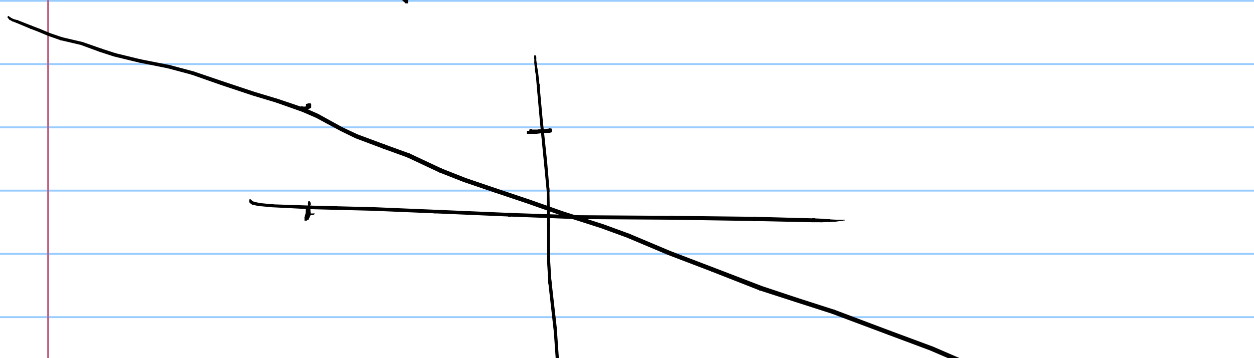
$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ Has no nontrivial null space

$$\begin{bmatrix} 2 & 1 \\ 1 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x + y = 0 \qquad y = -2x$$
$$x + \tfrac{1}{2} y = 0$$

$$x + \tfrac{1}{2}[-2x] = 0$$
$$\equiv 0$$

$$\begin{pmatrix} x \\ -2x \end{pmatrix} = x \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$



$N(A)$

Null space = set of all vectors $\vec{x}$, such that $A \cdot \vec{x} = 0$

Suppose that $\vec{x}, \vec{y} \in N(A)$

$$A \cdot \vec{x} = 0 \qquad A \cdot \vec{y} = 0$$

$$\vec{z} = \alpha \vec{x} + \vec{y} \qquad \text{is } \vec{z} \in N(A)?$$

$$A \cdot \vec{z} = A \cdot (\alpha \vec{x} + \vec{y}) = \alpha \underbrace{A \cdot \vec{x}}_{0} + \underbrace{A \cdot \vec{y}}_{0}$$

Null space is a linear space associated with A.

MMA - digression

Inverse[A]

RowReduce[A]

NullSpace[A]

NullSpace[{{2,1},{1, 1/2}}]

{-1/2, 1}

$\{\vec{x}_1, \vec{x}_2, \vec{x}_3, \ldots \vec{x}_n\}$ $\quad \bar{x}_i \in R^N$

$X \in R^{N \times n}$

$$X \cdot \vec{a} =$$

$$a_1 \vec{x}_1 + a_2 \vec{x}_2 + \cdots a_n \vec{x}_n$$

if $\boxed{X \cdot \vec{a} = 0}$

$$a_1 \vec{x}_1 + a_2 \vec{x}_2 \cdots + a_n \vec{x}_n = 0$$

if $a_1 \neq 0$ $\quad a_1 \vec{x}_1 = -a_2 \vec{x}_2 - a_3 \vec{x}_3 \cdots - a_n \vec{x}_n$

$X \cdot \vec{p} = \vec{q}$ suppose $\bar{p}$ solves

$$X \cdot (\vec{p} + \vec{a}) = X \cdot \vec{p} + \underbrace{X \cdot \vec{a}}_{0} = \vec{q}$$

$A \cdot \vec{x} = \vec{y}$ has a unique solution $\Longleftrightarrow$ there is no nontrivial null space $[N(A) = 0]$

$\Longleftrightarrow$ columns of A are linearly independent

$$\begin{array}{c} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \\ \\ \\ \\ \end{array} \begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \\ \text{———} \\ \text{———} \\ \text{———} \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\vec{x}$$

Row Space (A) = Span (Rows)

$\vec{y} \in R(A)$ $\qquad \vec{y} = \sum \alpha_i \vec{a}_i$

rows

$\vec{x} \in N(A)$ $\qquad \vec{y} \cdot \vec{x} = 0$

① $\qquad$ ②

$N(A) \perp R(A)$

any element in $N(A)$ is $\perp$
any element in $R(A)$

column space $R(A^T)$ ③

$N(A^T)$ ④ $\qquad$ if $\vec{y} \in N(A^T)$

$$A^T \cdot \vec{y} = 0$$

$$(A^T \cdot y)^T = y^T \cdot A$$

$$\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} & \end{bmatrix}\begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$