Physics 311, Fall 2005

Problem Set #8

Functions as vectors and a little Fourier analysis

Due: Wednesday, November 2

Quote of the Problem Set:
"The time you enjoy wasting is not wasted time." —Bertrand Russell

Read Chapter 3 §14 and Chapter 7 §§1-4, then do:

1. Chapter 3, §14, Problems 2, 4.

2. *Gram-Schmidt process for functions* Read carefully §14 example 6. You may learn later that the peculiar flavor of a given vector space of functions is imparted by the definition of its inner (scalar) product. Suppose we are given the non-orthogonal set of functions $\{f_0, f_1, f_2, f_3\} = \{1, x, x^2, x^3\}$ and the scalar product

$$\langle f_i | f_j \rangle \equiv \int_{-\infty}^{+\infty} dx\; f_i^*(x)\; f_j(x)\; e^{-x^2}, \tag{1}$$

find (following Eq. (14.10) of the the text) an orthonormal basis $\{p_0, p_1, p_2, p_3\}$. By all means use *Mathematica* to evaluate the scalar products and do the normalizations.

3. Chapter 3, §15, Problems 11, 12.

4. Chapter 7, §3, Problem 8.

5. *Useful* Show that

$$\int_{-L}^{+L} dx \sin\frac{n\pi x}{L} \sin\frac{m\pi x}{L} = L\delta_{n,m} \tag{2}$$

$$\int_{-L}^{+L} dx \cos\frac{n\pi x}{L} \cos\frac{m\pi x}{L} = L\delta_{n,m} \tag{3}$$

$$\int_{-L}^{+L} dx \sin\frac{n\pi x}{L} \cos\frac{m\pi x}{L} = 0. \tag{4}$$

Does anything unusual happen when $m = n = 0$?