PHGN 480: Laser Physics due 10 Oct 2008
Homework 3

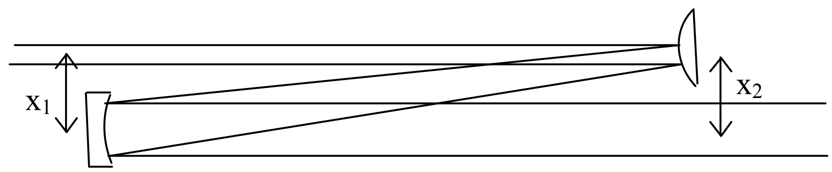
1) *Transition energies for iodine.* In the simplest, ideal model, the vibrational energies are given by $E_{vib} = \hbar\omega_{vib}\left(v+\frac{1}{2}\right)$, where $\hbar\omega_{vib} = 0.026$eV. The rotational energies are given by $E_{rot} = \hbar\omega_{rot} J\left(J+1\right)$, where $\hbar\omega_{rot} = 4.64\text{x}10^{-6}$ eV.
   a. Calculate the number of rotational levels in between vibrational levels.
   b. Make a plot of the distribution of population of rotational levels in the ground and first two vibrational states vs energy at room temperature. Here the 2J+1 degeneracy of the levels is important: the Boltzmann occupancy factor is (2J+1) exp(E/kT). This gives the range of initially-populated states for our absorption experiment.

2) *ASE threshold for amplifiers.* The maximum gain of an amplifier is limited by depopulation by amplified spontaneous emission (see section 2.9.2). A cylindrical rod of Nd:YAG with a diameter of 6.3mm (1/4 inch) and a length of 7.5cm is pumped very hard with a flashlamp such that the pump energy is uniformly distributed within the rod. Calculate the critical inversion for the onset of strong ASE, assuming there is no feedback from the boundaries of the rod. (The end faces are assumed to be perfectly anti-reflection coated.) Also calculate the maximum energy that can be stored in the rod to avoid the ASE. Data for Nd:YAG – peak cross-section for 1.064μm transition, $\sigma = 2.8\text{x}10^{-19}\text{cm}^2$, refractive index of YAG, n = 1.82.

3) *Gain saturation.* In a high-power amplifier, the beam passes through the gain medium once, and the beam size is adjusted to maximize the energy extracted from the amplifier.
   a. The saturated gain of a square pulse passing through an amplifier is given by the Franz-Nodvick equation (1963):
$$G = \frac{F_{sat}}{F_{in}} \log\left[1+\left(\exp\left(\frac{F_{in}}{F_{sat}}\right)-1\right)\exp\left(\frac{F_{stor}}{F_{sat}}\right)\right]$$
   Here, F_{sat} is the saturation fluence, F_{in} is the input fluence, and F_{stor} is the stored fluence (total extractable energy density times the length of the gain medium.) Show that in the limit of $F_{in}/F_{sat} \ll 1$, the single-pass gain reduces to the expected small-signal gain (independent of F_{in}).
   b. Calculate an expression for the ratio of the extracted energy to the stored energy. Plotting this versus input fluence for a few values of the small-signal gain. Explain the trend you see, and determine a good estimate for the value of F_{in}/F_{sat} that allows for extraction of 50% of the stored energy.
   c. The expression above assumes that electrons leave the lower lasing level in a time much shorter than the pulse duration. How would you expect your answer to change if the opposite were true?

   Comment: This ratio for input fluence/saturation fluence is the major determinant of how amplifiers are designed for different gain media. At one end, the saturation fluence for dyes is on the order of a few mJ/cm2, and at the other end, ytterbium amplifiers (e.g. Yb:KGW, Yb:glass) have a saturation fluence of around 10J/cm2. With low saturation fluence, the amplifier is easy to saturate, but there is a limited amount of energy that can be stored. With high saturation fluence, there is high energy storage, but it is difficult to extract the energy without damaging the laser rod.

4) *Astigmatism correction in a beam expander.* To expand a laser beam, it is often desirable to use curved mirrors instead of lenses, both to avoid extra dispersion and because the damage threshold for mirrors is higher than for lenses. The trouble is that tilting a curved mirror introduces astigmatism. A sketch of a system that avoids any intermediate focus is shown below.



The mirrors must be tilted to get the beam through, and a tilted mirror is astigmatic (the focal lengths in the horizontal and vertical planes are different). If θ is the incident angle (measured from the local surface normal to the center of the beam), the horizontal focal length (in the plane of the diagram) of a tilted mirror will be shortened: $f_H = f_0 \cos\theta$, where the normal focal length is f_0. The vertical focal length is lengthened to $f_V = f_0 / \cos\theta$.

Design a 5x beam expander that will take a collimated input (5mm diameter) and deliver a collimated output. The overall length should be no more than 300mm and there must be at least 25mm clearance between the center of the convex mirrors and the outgoing beam (distances x_2). As part of the design, determine a simple relationship between the ratios of the focal lengths f_1 and f_2 and the incident angles θ_1 and θ_2. You may assume that the incident angles are small, and that the mirror separation is close to the separation required for zero incident angle. You may use Mathematica if you need/want to: either with the lens imaging equation or with the ABCD matrices. Make a sketch of your final setup.

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