HW 7

Note Title 11/18/2006

Snieder 15.1

$\cos(n\pi x/L)$, $\sin(n\pi x/L)$

periodic with period $2L$

show

$f(x) = f(x+2L)$

i.e. $\cos\left(\frac{n\pi x}{L}\right) = \cos\left(\frac{n\pi(x+2L)}{L}\right)$

$= \cos\left(\frac{n\pi x}{L} + 2n\pi\right)$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\sin(2n\pi) = 0$

$\cos(2n\pi) = 1$

$\cos(n\pi x/L + 2n\pi) = \cos(n\pi x/L)$

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\sin(2n\pi) = 0$ so
$\cos(\quad) = 1$

$\sin(n\pi x/L + 2n\pi) = \sin(n\pi x/L)$

b) proof in Boas p351

c)

15.2

$f(x) = \frac{a_0}{2} + \sum a_n \cos(n\pi x/L) + b_n \sin(n\pi x/L)$

$\int_{-L}^{L} f(x)\sin(m\pi x/L)\,dx$ L.H.S.

RHS 1) $\int_{-L}^{L} \frac{a_0}{2} \underbrace{\sin(m\pi x/L)}_{\text{periodic on } -L, L}\,dx = 0$

2) $a_n \int_{-L}^{L} \cos(n\pi x/L) \sin(m\pi x/L)\, dx = 0$

by orthog.

3) $b_n \int_{-L}^{L} \sin(n\pi x/L) \sin(m\pi x/L)\, dx$

$= L\, \delta_{mn}$

$\Rightarrow \int_{-L}^{L} f(x) \sin(m\pi x/L)\, dx = b_m L$

$\Rightarrow b_m = \frac{1}{L} \int_{-L}^{L} f(x) \sin(m\pi x/L)\, dx$

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d) for $n=0$ $\cos(n\pi x/L) = 1$

so $\int_{-L}^{L} f(x)\, dx = \int_{-L}^{L} \frac{a_0}{2}\, dx$

$= 2L \frac{a_0}{2}$

$$\Rightarrow a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

for $n \neq 0$ arg. is ident. to sin case treated before.

e) f, g on $[-L, L]$: $(f \cdot g) = \int_{-L}^{L} f(x) g(x)\,dx$

so Let

$$u_n(x) = \frac{1}{\sqrt{L}} \sin(n\pi x/L)$$

$$(u_n, u_m) = \frac{1}{L}\int_{-L}^{L} \sin(n\pi x/L)\sin(m\pi x/L)\,dx$$

$$= \frac{1}{L}\left[L\, \delta_{mn} \right] = \delta_{mn}$$

f) if $f(x) = \sum c_n u_n(x)$

then

$$(f, u_m) = \sum_n c_n (u_n, u_m)$$

$$= \sum c_n \delta_{nm}$$

$$= c_m$$

$$f = \sum c_n u_n$$

$$\Rightarrow \quad f = \sum (f \cdot u_n) \quad u_n$$

$$\text{or} \qquad \sum u_n (u_n \cdot f)$$

g) $u_n(x) = \frac{1}{\sqrt{L}} \sin(n\pi x/L)$

$$u_n(x+\lambda) = \frac{1}{\sqrt{L}} \sin\left(\frac{n\pi(x+\lambda)}{L}\right)$$

$$= \frac{1}{\sqrt{L}} \left\{ \sin\left(\frac{n\pi x}{L} + \underbrace{\frac{n\pi\lambda}{L}}\right)\right.$$

$$\text{so } \lambda = \frac{2L}{n}$$

Boas

$$y = A \sin \frac{2\pi}{\lambda}(x - vt) \qquad \underline{2.10}$$

$$= A \sin 2\pi \left(\frac{x}{\lambda} - \underbrace{\frac{v}{\lambda}}_{v = \lambda/T} t\right)$$

$$= A \sin 2\pi \left(x/\lambda - t/T\right)$$

$$= A \sin \underbrace{\frac{2\pi}{\lambda} v}_{v/\lambda = f = \frac{\omega}{2\pi}} \left(\frac{x}{v} - t\right)$$

$$= A \sin \omega \left(x/v - t\right)$$

$$= A \sin\left(\frac{2\pi x}{\lambda} - \underbrace{\frac{2\pi v t}{\lambda}}_{\omega t = 2\pi f t}\right)$$

$$A \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

$\lambda = vT$

$$A \sin\left(\frac{2\pi x}{vT} - \frac{2\pi}{T} t\right)$$

$$= A \sin \frac{2\pi}{T}\left(\frac{x}{v} - t\right)$$

7.3.8

$$y = [A + B(\sin \omega t)](\sin \omega_c (t - x/v))$$

at a fixed distance from the source we can ignore the x dependence.

So $y(t) = [A + B \sin \omega t][\sin \omega_c t]$

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In[3]:= A[t_, w_, wc_] = Sin[(w - wc) t] + Sin[w t] + Sin[(w + wc) t] // FullSimplify
        A[t_, w_, wc_] = Cos[(w - wc) t] + Cos[w t] + Cos[(w + wc) t] // FullSimplify

Out[3]= (1 + 2 Cos[t wc]) Sin[t w]

Out[4]= Cos[t w] (1 + 2 Cos[t wc])
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It's easier to work backw. from the 3 frequencies. The choice of sin or cos doesn't matter.