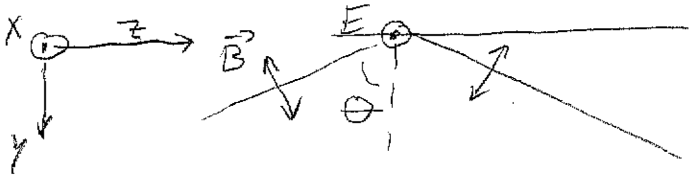
Parallel plane waveguide – conducting
work with $\vec{E}$ tangential, $\vec{B}^{\perp} \to 0$ at surface.
- perfect conductor has free charges + currents.
- this choice allows us to avoid working with them

free space waves are TEM: $\vec{E} \perp \vec{B} \perp \vec{k}$, all compon transve



TE: $\vec{B}$ has y, z compon.
$\vec{E}$ is only x
TM: $\vec{B}$ is transverse
solve for each case separately.

TE example:
construct solution from plane waves.

$$\vec{E}_+ = +\hat{x} E_0 e^{i(-k_y y + k_z z)} \quad \text{with} \quad k_y = k_0 \cos\theta$$
$$k_z = k_0 \sin\theta$$

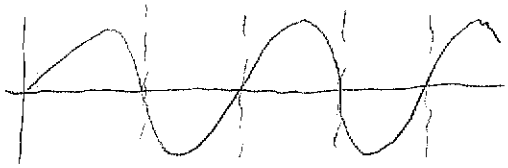
$$\vec{E}_- = -\hat{x} E_0 e^{i(+k_y y + k_z z)}$$

$\vec{E}_+ + \vec{E}_- = 0$ at $y = 0$

$$= \hat{x} E_0 e^{ik_z z}\left(e^{-ik_y y} - e^{+k_y y}\right) \quad 0 < y < b$$

$$= -2i E_0 \sin(k_y y)\, e^{ik_z z}$$

traveling wave in z
standing wave in y



could place second surface at any node w/o change to wave.

$\sin k_y b = 0 \qquad k_y b = k_0 b \cos\theta = n\pi \qquad n$ integer ≥ 1

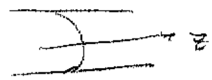
given λ or ω: allowed θ's $\qquad \cos\theta_n = \frac{n\pi}{k_0 b}$

given θ: allowed freq $\qquad \omega_n = \frac{n\pi c}{b\cos\theta}$

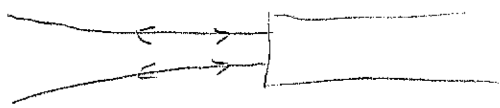
the wave now has a fixed profile in y direction
travels in z-direction:
$\vec{E} = \hat{x} E(y) e^{i(k_g z - \omega t)}$
with guide wavenumber $k_g = k_z = \sqrt{k_0^2 - k_y^2}$
discrete values of $k_y = n\pi/b$
$\rightarrow$ discrete values of k_g:

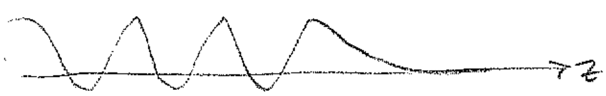
$$k_g = \sqrt{k_0^2 - (n\pi/b)^2} \quad \text{allowed values.}$$

cutoff frequency: express $k_0 = \omega/c$
$k_g = \sqrt{\omega^2/c^2 - (n\pi/b)^2}$
At $n=1$, transverse mode is:



at $\omega = \omega_c = \pi c/b$, cutoff frequency for $n=1$
$\rightarrow k_g = 0$
for $\omega < \omega_c$ $k_g \rightarrow$ pure imag., input wave reflects





can express cutoff in terms of $\nu_c = \frac{\omega_c}{2\pi} = \frac{nc}{2b}$
each mode index has a cutoff.

if input ω satisfies $\frac{\pi c}{b} < \omega < \frac{2\pi c}{b}$
guide allows only one mode.
"single-mode"

Propagation of waves in waveguide.

for simplicity, consider one transverse dimension

$$\vec{E}(y,z,t) = \hat{x} E_0 \sin\left(\frac{n\pi y}{b}\right) e^{i(k_z z - \omega t)}$$

phase velocity $\quad v_{ph} = \dfrac{\omega}{k_z}$

remember $\quad k_z = \sqrt{k_v^2 - k_y^2} = \sqrt{\omega^2/c^2 - \left(\frac{n\pi}{b}\right)^2}$

$$v_{ph} = \frac{\omega}{\sqrt{\omega^2 - \left(\frac{n\pi c}{b}\right)^2}} \cdot c > c$$

group velocity

$$v_g = \frac{d\omega}{dk_z} = \left(\frac{dk_z}{d\omega}\right)^{-1}$$

$$= \left[\frac{1}{2} \cdot \frac{2\omega}{c^2} \left(\frac{\omega^2}{c^2} - \left(\frac{n\pi}{b}\right)^2\right)^{-1/2}\right]^{-1}$$

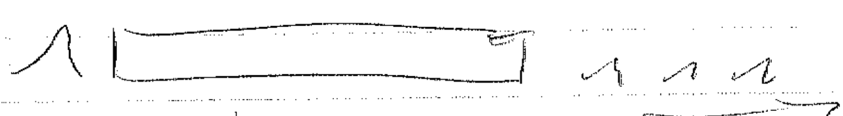
$$= \frac{c^2}{\omega} \cdot \frac{\omega}{c} \sqrt{1 - \left(\frac{n\pi c}{\omega b}\right)^2} \quad \text{less than } c$$

$$= c\sqrt{1 - \left(\frac{\omega_c^{(n)}}{\omega}\right)^2}$$

modal dispersion:

dependence of v_{ph}, v_g on mode index

multimode propagation:



each mode travels w/ diff't speed.

Metal Waveguides

including vector character and boundary condi
vectors: anticipate orientation of fields rel. to walls.
work solution

TEM $\vec{E} \perp \vec{B} \perp \hat{z}$ $E_z = B_z = 0$
example: free space plane wave.

TE $\vec{E}$ is transverse only $\vec{E}_z = 0$

$B_z \neq 0$

TM $B_z = 0$

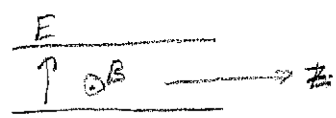
"transverse" means field is $\perp \hat{z}$

Remember, once modes are known (eigenfunctions)
an arbitrary solution can be composed

$$\vec{E} = \sum_m a_m E_{TE_m} + \sum_l a_l E_{TM_l}$$

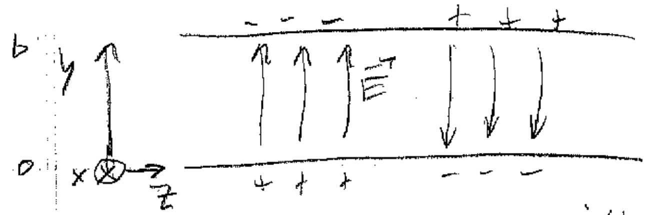
modes are a complete set that can describe
any function compatible with boundary conditions.

Strip lines: plane parallel conductors
assume perfect conductor → surface charge, current.
TEM is allowed:



same velocity as in vacuum (or air)

no modes.



For TEM $\vec{E} = \hat{y} E_0 e^{i(kz-\omega t)}$ along $\hat{x}$ $E \parallel$ to metal is zero.

since $\nabla \cdot \vec{E} = 0$ inside wg,

$\rightarrow \frac{\partial E}{\partial y} = 0$ $\quad$ $\vec{E}$ is constant

finite value of $E_\perp$ at metal $\rightarrow$ surf. charge.

- plates are not connected electrically.

get $\vec{B}$ from $\vec{E}$:

$$\nabla \times \vec{E} = i\frac{\omega}{c}\vec{B} \qquad E \text{ varies only w/ } z.$$

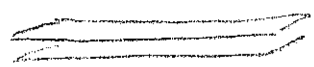
$$-ikE_0\hat{x} = i\frac{\omega}{c}\vec{B} \qquad \begin{array}{ccc} x & y & z \\ \partial_x & \partial_y & \partial_z \\ & E_y & \end{array}$$

$$B = -\hat{x}E_0 \qquad \omega = kc$$

from wave eqn.

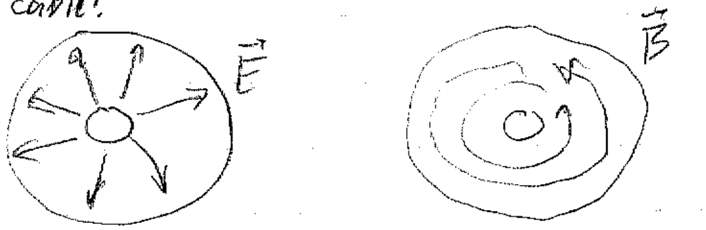
Wave behaves similar to free wave, but $\vec{E}$ only along y, $\vec{E}$ is confined btw. plates.

in practice: striplines



- no cutoff ∴ prop. long λ in small guide.

coax. cable:



wave velocity determined by dielectric betw. conductors.

TE modes

$\vec{E}$ is $\perp$ propagation direction: either $\hat{x}, \hat{y}$

$\nabla \cdot \vec{E} = 0 \qquad E_z = 0$ and $\partial_x = 0$

$\rightarrow \partial_y E_y = 0 \quad \rightarrow E_y =$ const. or 0

$\searrow$ TEM $\qquad \searrow$ TE

$\therefore \ \vec{E} = \hat{x} E_x(y) e^{i(k_z z - \omega t)}$

soln for $\vec{E} \rightarrow \vec{B}$ thru $\nabla \times \vec{E} = -\frac{1}{c}\partial_t \vec{B} = \frac{i\omega}{c}\vec{B}$

$$\begin{vmatrix} x & y & z \\ \partial_x & \partial_y & \partial_z \\ E_x(y) & 0 & 0 \end{vmatrix} \qquad \hat{y}\,\partial_z E_x - \hat{z}\,\partial_y E_x = i\frac{\omega}{c}\vec{B}$$

two components of $\vec{B}$

since $E_x(0) = E_x(b) = 0$

$\rightarrow E_x(y) = E_0 \sin\left(\frac{n\pi y}{b}\right) \qquad \rightarrow k_{yn} = n\pi/b$