

(c) The dispersion relation is $\left(\frac{\omega}{b}\right)^2 = \omega^2/c^2 - k^2$

We get the phase velocity from

$$v_{ph} = \frac{\omega}{k} = \frac{\omega}{\sqrt{\omega^2/c^2 - \left(\frac{\omega}{b}\right)^2}} = c \cdot \frac{\omega}{\sqrt{\omega^2 - (\omega bc/b)^2}}$$

So $v_{ph} > c$ always, unless $\omega > \frac{\omega bc}{b}$

where things break down. We identify this as the cutoff frequency

The group velocity comes from:

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk} \left(\sqrt{\left(\frac{\omega bc}{b}\right)^2 + k^2 c^2} \right) = \frac{1}{2} \left(\left(\frac{\omega bc}{b}\right)^2 + k^2 c^2 \right)^{-1/2} \cdot 2kc^2$$

$$= \frac{kc^2}{\sqrt{\left(\frac{\omega bc}{b}\right)^2 + k^2 c^2}} = c \cdot \frac{k}{\sqrt{\left(\frac{\omega bc}{b}\right)^2 + k^2}}$$

Which is certainly less than c . Subbing in for $k + k^2$ in terms of ω , we can also express v_g as

$$v_g = c \sqrt{1 - \left(\frac{\omega bc}{\omega b}\right)^2}$$