

To get full credit, you must show all of your work.

1. Solve the following differential equation explicitly:  $\frac{dy}{dt} = \frac{t}{3y + t^2y}, y \neq 0$

2. Solve the following IVP explicitly:  $\frac{1}{y} dy + ye^{\cos(t)} \sin(t) dt = 0, y\left(\frac{\pi}{2}\right) = 1, y \neq 0$

3. There is a population,  $P$ , of bacteria living in a Petri dish, which is known to increase at a rate proportional to the number of bacteria present at any time,  $t$ .
  - a. What is the differential equation that models this situation? Is this equation linear?
  - b. Find the solution to your differential equation from part (a): If the population has doubled after 10 hours, then how long will it take to triple?
  - c. Now assume that a poison is dropped into the Petri dish. The rate at which the bacteria is being killed by the poison is  $.001P^2$ . Change the differential equation from part (a) to take into account this new factor, but do not solve this new ODE. Is this equation linear?
  - d. For each ODE from parts (a) and part (c), find all the equilibrium solutions, in other words when  $\frac{dP}{dt} = 0$ . When applicable leave your answer in terms of the growth constant,  $k$ .
4. Find an example of a differential equation from a field of study that is of interest to you. Write the ODE in the space below. Describe the variables and what it models.

5. You are preparing a large pot of stew that will be fed to hungry college students the next day. When it is finished cooking, the stew is  $120^{\circ}\text{F}$ , but it cannot be refrigerated until the stew has cooled to  $60^{\circ}\text{F}$ . By cooling the pot in a sink full of cold water, (kept running, so that its temperature was roughly constant at  $35^{\circ}\text{F}$ ) and stirring occasionally, the temperature of the stew drops to  $105^{\circ}\text{F}$  in ten minutes. How long will it take for the stew to reach a temperature appropriate for refrigeration? Assume that the rate of cooling is proportional to the difference between the current temperature and the ambient temperature. (Leave your answer in terms of natural logs.)

6. Using the software that came with your book, graph the slope field for the differential equation:  $\frac{dy}{dt} = \cos(ty) + y - t$  with the bounds of  $[-5,5]$  on both  $y$  and  $t$ . This software is also available at <http://alamode.mines.edu/bdh.html>. Print out the page and attach it to your worksheet. Hint: you may want to use HPGSolver.

7. Match the following differential equations to their slope fields:

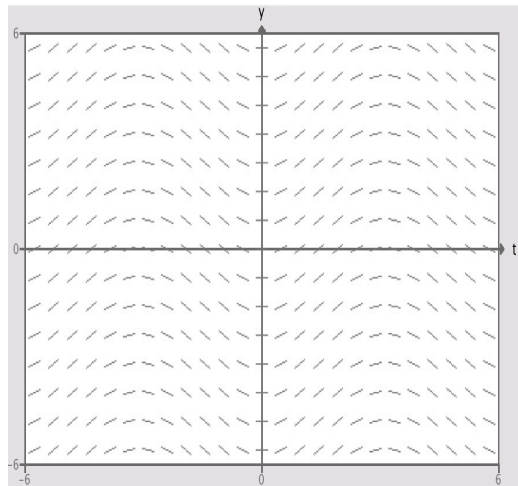
a.  $\frac{dy}{dt} = t^2 - y$  \_\_\_\_\_

b.  $\frac{dy}{dt} = \sin(y)$  \_\_\_\_\_

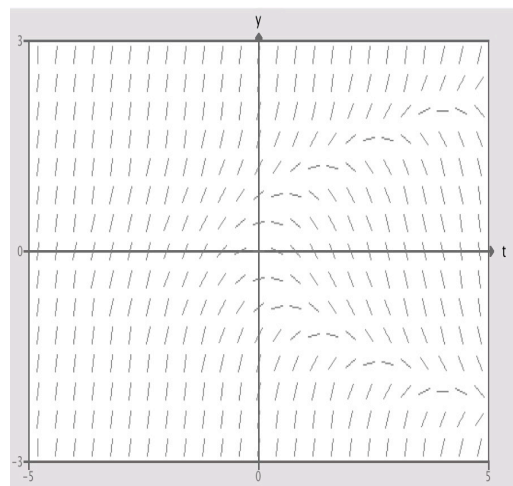
c.  $\frac{dy}{dt} = \sin(t)$  \_\_\_\_\_

d.  $\frac{dy}{dt} = y^2 - t$  \_\_\_\_\_

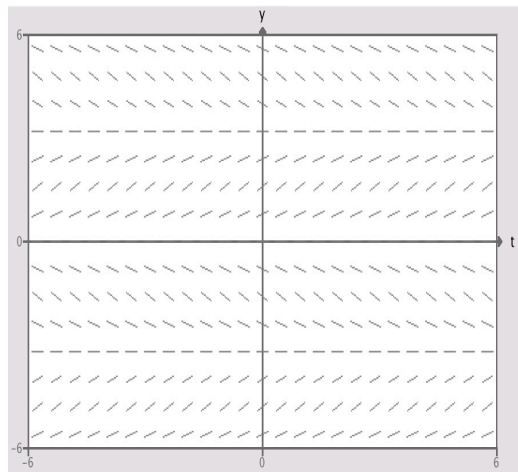
I.



II.



III.



IV.

