Reading Today: 9.4
Tomorrow: 9.5

Absorption and Dispersion.

We've delt w/ transparent material where μ, ϵ are constant.

First, let's talk about absorbing materials (non-transparent).

Examples: metals {conducting material}

Let's look at what happens with some free charge.

Maxwell's eqns become

(i) $\vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\epsilon}$ (iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(ii) $\vec{\nabla}\cdot\vec{B} = \phi$ (iv) $\vec{\nabla}\times\vec{B} = \mu\vec{J}_f + \mu\epsilon\frac{\partial \vec{E}}{\partial t}$

Ohm's Law! $\vec{J} = \underset{\substack{\uparrow \\ \text{conductivity.}}}{\sigma}\vec{E}$ $\sigma = \frac{1}{\underset{\substack{\uparrow \\ \text{resistivity}}}{\rho}}$

For $\rho_f, \vec{J}_f \Rightarrow \vec{\nabla}\cdot\vec{J}_f = -\frac{\partial \rho_f}{\partial t}$

$$= \vec{\nabla}\cdot(\sigma\vec{E}) = -\frac{\partial \rho_f}{\partial t} = \sigma\vec{\nabla}\cdot\vec{E} = -\frac{\partial \rho_f}{\partial t}$$

$$\Rightarrow \frac{\sigma}{\epsilon}\rho_f = -\frac{\partial \rho_f}{\partial t} \Rightarrow \rho_f = \rho_{f0}e^{-\sigma/\epsilon\, t}$$

For steady-state problems, we take $\rho_f = \phi$

$\vec{J}_f = \sigma\vec{E} \Rightarrow$ Max(4) $\Rightarrow \vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\epsilon\frac{\partial \vec{E}}{\partial t}$

Take the $\vec{\nabla}\times$(Max(3)), eliminate $\vec{B}$, to find the new wave eqn. for $\vec{E}$.

$$\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t} + \mu\sigma\frac{\partial\vec{E}}{\partial t}; \quad \nabla^2\vec{B} = \mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2} + \mu\sigma\frac{\partial\vec{B}}{\partial t}$$

General solution for a wave in the z direction is $\tilde{\vec{E}} = \vec{E}_0 e^{-Kz} e^{i(kz-\omega t)} = \vec{E}_0 e^{i[(k+iK)z-\omega t]}$

$= \vec{E}_0 e^{i(\tilde{k}z-\omega t)}$ $\quad \tilde{k} = k + iK$

Show this is a solution and solve for $\tilde{k}^2$ in terms of $\mu, \epsilon, \omega, \sigma$.

$\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$

(a) Show we can recast our dissipative wave eqn. as $\nabla^2\vec{E} = \mu\tilde{\epsilon}\frac{\partial^2\vec{E}}{\partial t^2}$; solve for $\tilde{\epsilon}$ $[\mu, \epsilon, \omega, \sigma]$

(b) Assuming harmonic time dependence, show $\vec{\nabla}\times\vec{B} = \mu\tilde{\epsilon}\frac{\partial\vec{E}}{\partial t}$

$\tilde{\epsilon} = \epsilon + \frac{i\sigma}{\omega}$

Maxwell's equations one more time

(i) $\vec{\nabla}\cdot\vec{E} = \phi$ $\quad$ (iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

(ii) $\vec{\nabla}\cdot\vec{B} = \phi$ $\quad$ (iv) $\vec{\nabla}\times\vec{B} = \mu\tilde{\epsilon}\frac{\partial\vec{E}}{\partial t}$

Assuming transient ρ_f is gone.

<u>Boundary conditions</u> <u>from</u> <u>Book</u>

$\epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = \sigma_f$ $\quad$ $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$

$B_1^\perp = B_2^\perp$ $\quad$ $\frac{1}{\mu_1}B_1^{\parallel} - \frac{1}{\mu_2}B_2^{\parallel} = \cancel{\vec{K}_f}^{\,\phi} \times \hat{n}$

If $\vec{J} = \sigma\vec{E}$, and $\vec{K} \neq \phi$, $\Rightarrow \vec{J} = \infty$ on surface $\Rightarrow \vec{E} = \infty$.

$\therefore \vec{K}$ must $= \phi$.

It turns out, using $\tilde{\epsilon}$ {$\vec{E}$'s, $\vec{B}$'s now complex generalizations}

$\tilde{\epsilon}_1 E_1^\perp = \tilde{\epsilon}_2 E_2^\perp$ $\quad$ $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$

$B_1^\perp = B_2^\perp$ $\quad$ $\frac{1}{\mu_1}\vec{B}_1^{\parallel} = \frac{1}{\mu_2}\vec{B}_2^{\parallel}$

Often in references, rather than $\tilde{\epsilon}$ (ϵ_r, ϵ_I), they quote

$\sqrt{\tilde{\epsilon}/\epsilon_0} = n \pm ik$

Physical consequences!

Loss: Assume we have an absorbing medium w/ normally incident light on it.

If it's thick $T = \emptyset$.

How much is lost $= 1 - R$

$$R = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)\left(\frac{1-\tilde{\beta}^*}{1+\tilde{\beta}^*}\right) \quad \text{where } \tilde{\beta} = \sqrt{\frac{\mu_1 \tilde{\epsilon}_2}{\mu_2 \tilde{\epsilon}_1}}$$

Assuming $\tilde{\epsilon}_1 = \epsilon_1$; $\mu_1 = \mu_2 = \mu_0$

$$\tilde{\beta} = \sqrt{\frac{\tilde{\epsilon}_2}{\epsilon_1}} \qquad \tilde{\beta} + \tilde{\beta}^* = 2\,\mathrm{Re}[\tilde{\beta}]$$

$$R = \frac{1 - \tilde{\beta} - \tilde{\beta}^* + |\beta|^2}{1 + \tilde{\beta} + \tilde{\beta}^* + |\beta|^2} = \frac{1 - 2(\mathrm{Re}[\tilde{\beta}]) + |\tilde{\beta}|^2}{1 + 2\,\mathrm{Re}[\tilde{\beta}] + |\tilde{\beta}|^2}$$

$$R = 1 - \frac{4(\mathrm{Re}[\tilde{\beta}])}{1 + 2\,\mathrm{Re}[\tilde{\beta}] + |\tilde{\beta}|^2}$$

$$\text{Loss} = 1 - R = \frac{4(\mathrm{Re}[\tilde{\beta}])}{1 + 2\,\mathrm{Re}[\tilde{\beta}] + |\tilde{\beta}|^2}$$