E. Kreyszig, Advanced Engineering Mathematics, $8^{\text {th }}$ ed.

## Lecture: Overview and Outline Module: 00

Suggested Problem Set: \{NULL\}
August 11, 2008

## Quote of Lecture 0

Professor Hubert Farnsworth: Good news, everyone. You'll be making a delivery to the planet Trisol. A mysterious planet located in the mysterious depths of the Forbidden
Zone.
Leela: Professor, are we even allowed in the Forbidden Zone?

Professor Hubert Farnsworth: Why of course. It's just a name, like the Death Zone, or the Zone of No Return. All the zones have names like that in the Galaxy of Terror.

Futurama: My Three Suns (1999)
The course Advanced Engineering Mathematics serves the following CSM disciplines,

- Engineering (Civil, Electrical, Environmental, Mechanical)
- Geophysics
- Rouge Physicists
- Students pursuing an ASI or minor in Mathematics
by introducing them to concepts from,
- Linear Algebra
- Partial Differential Equations
in order to connect their two-years of post secondary mathematics to the rich field of applied mathematical modeling.

If mathematics is the study of the meaning and properties associated with the symbolic formalism then applied mathematical modeling is the application of this knowledge to real-world phenomenon in an effort to draw non-experimental conclusions. The ultimate goal is to make predictions about natural occurrences in order to gain control over them. ${ }^{1}$ Since this has been going on for most of human existence the body of material is massive and deep. We will be mostly concerned with calculation, but if we remember to also concern ourselves with the mathematical roots we will achieve a more comprehensive and thus connected understanding enabling us to remember more concepts for a longer period of time.

It is my perspective that the key point of this material is to draw as many conclusions about the symbols, which naturally arise as solutions to certain differential equations, as possible. This is by no means a small task and there are many different and equally justifiable routes to this goal. However, this goal, I feel, is best served by studying first the straight-forward concepts of linear algebra and connect these concepts to the more complicated study of linear partial differential equations used to model ideal, flows, vibrations, and potential fields. ${ }^{2}{ }^{3}$

[^0]My opinion is largely due to the fact that linear algebra can abstract a method, which you should already be aware of, to any finite number of directions. That is, for certain two-dimensional linear problems say,

$$
\frac{d \mathbf{Y}}{d t}=\left[\begin{array}{ll}
a & b  \tag{1}\\
c & d
\end{array}\right] \mathbf{Y}, \quad \mathbf{Y}(0)=\left[\begin{array}{l}
x_{0} \\
y_{0}
\end{array}\right]
$$

one can construct solutions via linear combinations in some eigenbasis of the constant coefficient matrix. Specifically, solutions takes the form,

$$
\begin{equation*}
\mathbf{Y}(t)=k_{1} \mathbf{v}_{1} e^{\lambda_{1} t}+k_{2} \mathbf{v}_{2} e^{\lambda_{2} t} \tag{2}
\end{equation*}
$$

for appropriate choices of $k_{1}, k_{2}, \lambda_{1}, \lambda_{2}, \mathbf{v}_{1}, \mathbf{v}_{2}$. ${ }^{4}$ This concept is deep and difficult to pick out when one is constructing solutions in infinite-dimensional spaces. ${ }^{5}$ So, we start with the a study of linear algebra, building on your previous work with differential equations and vector calculus, so that when we finish with a survey of PDE's the mathematics will have a better 'sense'.

For example, we will find out later on that for solutions to the linear PDE,

$$
\begin{equation*}
\frac{\partial u}{\partial t}=c^{2} \triangle u \tag{3}
\end{equation*}
$$

whose unknown function $u$ has spatial component defined on a closed and bounded domain in $\mathbb{R}$, can take the following form,

$$
\begin{equation*}
u(x, t)=\sum_{n=-\infty}^{\infty} k_{n} e^{-i \omega_{n} x} e^{-\left(c \omega_{n}\right)^{2} t} \tag{4}
\end{equation*}
$$

for appropriate choice of $k_{i}, w_{i}, i \in \mathbb{N}$. It is not obvious what this summation means and how it could possibly be related to the study of the PDE, which models the flow of some density. It may or may not be obvious that this is a linear combination of basis vectors, or that since we used infinitely many basis vectors the summability of the series should be in question. ${ }^{6}$ However, if we start small and go big, we will have the experience needed to be comfortable with statements like (4) and be able to concentrate on understanding what they can tell us about (3). In the past I have not delivered the material in this way.

Traditionally, my course starts with the topic of Fourier series and builds to PDE's. After this was completed the course would end with the reprieve of linear algebra. It is understandable why everyone found this timeline so amenable. However, this tends to make the hardest calculations encountered in the study of PDE's shadowy and thus difficult to understand and replicate. Breaking with these previous timelines I offer, a hopefully beneficial, ordering outlined below. I ask each student to read this along with your texts table of contents in order to note what I feel are key concepts and where/when we expect them to come up.

[^1]
## MATH348-Fall2008 - Tentative Schedule

| Lecture(s) | Section | Pages | Key Concepts |
| :---: | :---: | :---: | :---: |
| 1 | 7.1,7.2 | 272-286 | Algebra, Associativity, Commutativity, Distribution, Inner-Product, Outer-Product, Matrix Product, Symmetric, Skew-Symmetric |
| 2-4 | 7.3,7.5 | $\begin{aligned} & 287-295, \\ & 302-305 \end{aligned}$ | Linear System, Existence and Uniqueness, Gauss Elimination, Row Echelon Form, Fundamental Theorem for Linear Systems, Homogeneous and Nonhomogeneous systems. |
| 6-7 | 7.7-7.8 | 308-314 | Determinant, Cramer's Theorem, Matrix Inverse, Orthogonal Matrix |
| 8-10 | 7.4, 7.9 | $\begin{aligned} & 296-301, \\ & 323-329 \end{aligned}$ | Linear Dependence, Basis, Dimension, Rank, Span, Row Space, Column Space, Null Space, Vector Space, Inner Product Space |
| 11 | 8.1 | 334-339 | Eigenvalue, Spectra, Eigenvector, Eigenfunction |
| 12 | 8.3 | 345-348 | Symmetric, Skew-Symmetric, Orthogonal, Transformations, Spectra |
| 13-14 | 8.4 | 349-355 | Eigenbasis, Diagonalization, Quadratic Form, Definiteness |
| 15-16 | Review of Functions | N/A | Function, Even, Odd, Periodic Function, Trigonometric Function, Factorial Function, Gamma Function, Bessel Function of the First Kind |
| 17-18 | 11.1, 11.3 | $\begin{aligned} & 478-486, \\ & 490-495 \end{aligned}$ | Fourier Series, Fourier Coefficents, Fourier Series of Functions with Symmetry |
| 18 | 11.2 | 487-489 | Domain Scaling Properties |
| 19 | 11.4 | 496-498 | Euler's Formula, Complex Fourier Series |
| 20 | 11.6 | 502-505 | Trigonometric Approximation |
| 21 | 11.7-11.8 | 506-517 | Fourier Integral, Fourier Sine/Cosine Transform |
| 22-25 | 11.9 | 518-528 | Fourier Transform, time/space domain, frequency domain, spectral representation, convolution, Green's function, Frequency Response |
| 26 | $\begin{aligned} & \text { Review of DE, } \\ & 12.1 \end{aligned}$ | 535-537 | Differential Equation, Vocabulary, Linear ODE's, Boundary Value Problems, Simple Harmonic Oscillators, Bessel's Equation |
| 27-28 | Flows and Conservations Laws | N/A | Divergence Theorem, Conservation Equation, Constitutive Equation, Fourier's Law of Heat Conduction |
| 29 | 12.5 | 552-561 | Boundary Conditions, Separation of Variables, Periodic Extension |
| 30 | Inhomogeneity | N/A | Extension of Fourier Methods |
| 31 | 12.2-12.4 | 538-551 | Ideal Wave Equation, Vibrations, D'Alebert's Solution |
| 32 | 12.6 | 562-568 | Cauchy-Problem, Heat Kernel |
| 33 | 12.9 | 579-586 | Multivariate Chain Rule, Laplacian in Polar Coordinates, Fourier-Bessel Series |
| 34 | 12.10 | 587-593 | Cylindrical and Spherical Geometries |
| 35 | 12.11 | 594-596 | Laplace Transforms and PDE's |
| 36 | Acoustics | N/A | Linear Approximations and Small Amplitude Vibrations |


[^0]:    ${ }^{1}$ For example, it is sometimes difficult to construct experiments involving complicated fluid flow but it is 'easier' to write down mathematical models for these flows and evaluate them under 'experimental conditions'. A reason for this might be to understand how to mix two fluids into one fluid while using the least amount of energy.
    ${ }^{2}$ I must note that it is highly useful to study the theory of linear algebra as it is the most applicable mathematical tool in the sciences and well worth a stand alone course. Those interested should consider taking MATH332-Linear Algebra.
    ${ }^{3}$ The concept of linearity is a powerful tool that allows one to say that an object with a certain property is equivalent to the sum of of other objects having the same property. This is not generally true of nonlinear phenomenon.

[^1]:    ${ }^{4}$ You may recall that these values are determined by the initial conditions, eigenvalues and eigenvectors, respectively.
    ${ }^{5}$ In the previous problem the solution space has a two-dimensional basis and thus all solutions to the problem can be written using linear combinations of these two eigenvectors. This is analygous to the concept that any vector in the plane can be written as the linear combination of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.
    ${ }^{6}$ We won't question it here and for many the theory is not needed for the fearless calculations. However, we should at least make a mental note that we will be on slippery slope. One can show that there is a convergent trigonometric series that is not the Fourier series of any integrable function and at some point we will construct a series used to represent a function, which is allowed be called equivalent even though we permit it to differ from the function at a countably-infinite amount of points, bringing into question what we actually mean by integral.

