Free spectral range + resolution in Fabry-Perot interferometers.

transmission: $T = \left(1 + F\sin^2(\delta/2)\right)^{-1}$

peak transmission at $\sin^2(\,) \to 0$

$\delta/2 = m\pi \quad \to \quad \frac{1}{2}\frac{2\pi}{\lambda_0}\cdot 2nd\cos\theta_t = m\pi$

$nd\cos\theta_t = m\lambda_0/2$ integer # of $\lambda_0/2$

express spacing in terms of frequency:

$\nu = c/\lambda$

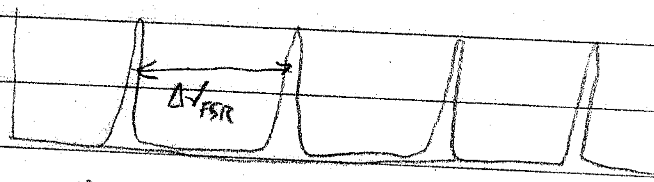
$\nu_m \frac{2nd}{c}\cos\theta_t = m$ → equal spacing in ν

consider neighboring orders $m, m+1$

$\nu_{m+1} - \nu_m = \Delta\nu = \dfrac{c}{2nd\cos\theta_t}$

At $\theta = 0$,

$\Delta\nu = \dfrac{c}{2nd} \equiv \Delta\nu_{FSR}$ note $\Delta\nu_{FSR} = 1/\tau_{RT}$ round trip time.



ex. $d = 5\,mm$, $n = 1$

$\Delta\nu = c/2nd = 3\times10^{10}\,cm/s / 2\cdot0.5 = 30\,GHz$

what is $\Delta\lambda$?

> remember $\Delta\lambda/\lambda = \Delta\nu/\nu = \Delta\omega/\omega$

$\Delta\lambda = \dfrac{\lambda_0^2}{c}\Delta\nu$ at $\lambda_0 = 500\,nm$

$\Delta\lambda = 0.025\,nm$

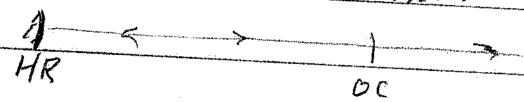
$\nu = c/\lambda \to \dfrac{d\nu}{d\lambda} = -\dfrac{c}{\lambda^2}$

$\to \dfrac{d\nu}{\nu} = -\dfrac{d\lambda}{\lambda}$

Sodium D doublet $\Delta\lambda \approx 0.6\,nm$ $\lambda = 590\,nm$

→ $d < 290\,\mu m$ must have $\Delta\nu_{FSR}$ large enough to ensure neighboring pks are of same order.

Applications: étalon to make single mode laser.

laser cavity 

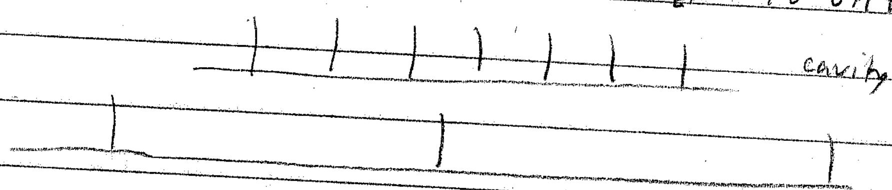
passive cavity = FP

$L = 15\,cm \quad n = 1 \rightarrow \Delta\nu_c = c/2L = 1\,GHz$.

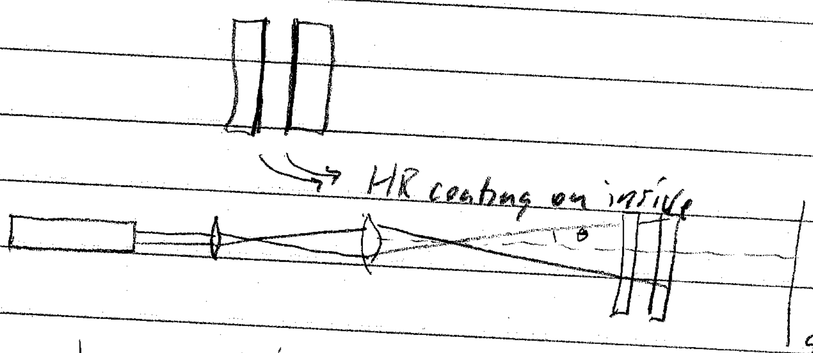
HeNe gain covers several of these lines. say 5.

$\therefore$ insert solid FP (= etalon), angle tune to select one line, = one longitudinal mode.

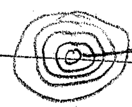
$d = 1\,cm \quad n = 1.5 \rightarrow \Delta\nu_{Et} = 10\,GHz$.



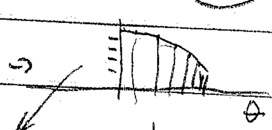
Interferometer:



here, radial position → incident angle. $\theta_t = \theta_i$

→ circular fringes  → start at m = large # = m_0

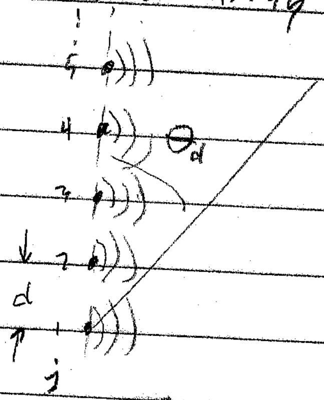
$\Delta m = 1$

Since $\cos\theta \rightarrow$  $\cos\theta_m = \dfrac{m\lambda_0}{2nd}$

equal spacing in $\cos\theta$ tighter spacing in θ

Diffraction gratings: geometric analysis (interference)

Consider array of point sources:



all sources start w/ same phase.
observe at large distance such that each pt source → plane wave.

$\Delta\phi = k_0 n d \sin\theta_d$

$$E = \sum_{j=1}^{N} E_j = E_1 \left(1 + e^{i\Delta\phi} + e^{i2\Delta\phi} + \dots e^{i(N-1)\Delta\phi}\right)$$

sum series $\displaystyle\sum_0^{N-1} r^j = \frac{1-r^N}{1-r}$ for $|r|<1$

$$\to \frac{e^{iN\Delta\phi}-1}{e^{i\Delta\phi}-1} = \underbrace{\frac{e^{iN\Delta\phi/2}}{e^{i\Delta\phi/2}}}_{e^{i\frac{(N-1)}{2}\Delta\phi}} \frac{\sin(N\Delta\phi/2)}{\sin(\Delta\phi/2)}$$

= phase shift from center of array to edge.

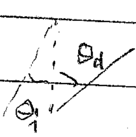
Peaks at $\frac{\Delta\phi}{2} = m\pi$ where $\sin\Delta\phi/2 = 0$

at these peaks sum → N. $E \propto N$, $I \propto N^2$

$\to m\lambda_0 = nd\sin\theta_d$

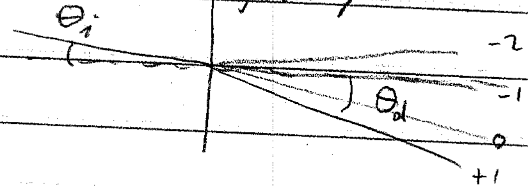
m = diffraction order
d = groove spacing

non-normal incidence



$\to m\lambda_0 = nd(\sin\theta_d - \sin\theta_i)$

transmission grating:

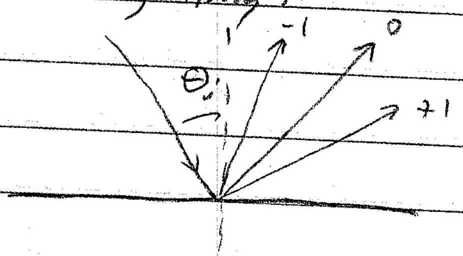


at zero order $m = 0$

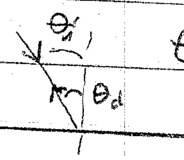
$\theta_d = \theta_i$

$m = 1 \quad \theta_d > \theta_i$

reflection grating:



Littrow: $m = -1$



$\theta_i = -\theta_d$

Angular dispersion: $\mathcal{D} = d\theta_m / d\lambda$

for $n=1$, $d(\sin\theta_m - \sin\theta_i) = m\lambda$

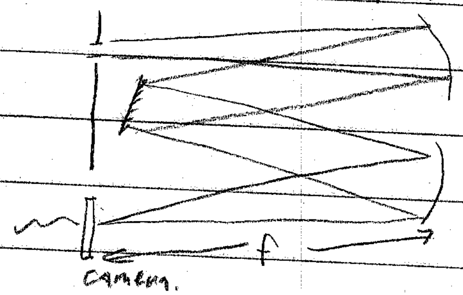
$\cos\theta_m \left(d\theta_m / d\lambda \right) = m/d$

$\mathcal{D} = m / d\cos\theta_m$

greater angular dispersion w/ large m, small d

large θ_m (near grazing)

spectrometer construction:



rotate grating to direct diff't λ's to exit.

At exit $\Delta\theta = \Delta x / f$

$$\frac{d\lambda}{dx} = \frac{1}{(dx/d\theta)(d\theta/d\lambda)} = \frac{d\cos\theta_m}{mf}$$

example $1/d = 1200$ gr/mm ($d = 833$ nm)

Littrow angle: $\sin\theta_i = \lambda/2d$ at 500 nm $\theta_i = 17.5°$

For $f = 0.5$ m $\left|\frac{\Delta\lambda}{\Delta x}\right| = \frac{1.6 \text{ nm}}{\text{mm}}$ if slit is 10 μm

$\rightarrow$ min $\Delta\lambda = 0.016$ nm.

what is actual resolution? do diffraction calc.