300-01 Homework 1.0 **Elvis Presley**

2. Your first LaTeX assignment is to use LaTeX to produce a document that replicates this as exactly as possible, with just two differences: First, replace the name above with your own. Second, make the following letter substitutions so that I know that you did not just photocopy this document:

- Problems 3 and 5, change each m to n;
- Problem 8, change each c to b.

Your grade on this assignment will be based on how much your paper looks like this one.

3. Prove that every integer that is divisible by 6 is even.

Proof. Suppose $m \in \mathbb{Z}$ is divisible by 6. Then there is some $k \in \mathbb{Z}$ such that $m = 6k$. Therefore $m = 2(3k)$, and since $3k$ is also in $\mathbb{Z}$, this means that m is divisible by 2 and therefore that m is even. $\square$

5. Define $A = \{m \in \mathbb{Z} \,|\, m^3 - m^2 - 6m = 0\}$. Prove that if $m \in A$ then $m = -2, 0,$ or 3.

Proof. Let $A = \{m \in \mathbb{Z} \,|\, m^3 - m^2 - 6m = 0\}$. Note that

$$\begin{aligned} m^3 - m^2 - 6m &= m(m^2 - m - 6) && \text{(factor out an } m\text{)} \\ &= m(m+2)(m-3). && \text{(factor the quadratic)} \end{aligned}$$

Therefore if $m \in A$ then $m(m+2)(m-3) = 0$, and therefore m must be equal to one of -2,0, or 3. $\square$

8. Prove that if $a, c \in \mathbb{R}$ with $a \leq c$ then $[c, \infty) \subseteq [a, \infty)$.

Proof. Suppose $a \leq c$ in $\mathbb{R}$. For all $x \in \mathbb{R}$,

$$\begin{aligned} x \in [c, \infty) &\Longrightarrow x \geq c \\ &\Longrightarrow x \geq c \geq a && (c \geq a \text{ by hypothesis}) \\ &\Longrightarrow x \geq a && \text{(transitivity)} \\ &\Longrightarrow x \in [a, \infty). \end{aligned}$$

Therefore we have $[c, \infty) \subseteq [a, \infty)$. $\square$