Jan 16 — Sec 1-4 & 2-4 & conservation of charge

(1) Gauss's Law

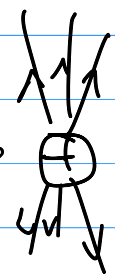
$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0}$$

Incongruous: Doesn't it work only for symmetric shapes?

Congruous: How is it applied to an egg shaped surface?

How does it relate to the divergence theorem?

Modifying: Moving charge does Gauss's law still hold?



How does medium effect gauss's law?

Generalizing/Analogy: Is there a Gauss's law for gravity?

Causal/Creative: Why does it work?

Informational: Why does it need a closed surface?

Any size restrictions on it application?

(2) Conservation of charge

Expressed as a PDE using flux (chapter 9 in Shadowitz for conductors)

Flow of charge thru a surface

$$= \underbrace{\int \vec{J} \cdot d\vec{a}}_{\text{flux: } \frac{\text{Coulombs}}{s}} \qquad \vec{J} \equiv \rho \vec{v} \quad \frac{\text{Coulombs}}{m^3}\frac{m}{s} = \frac{C}{s}\frac{1}{m^2}$$

$$\oint \vec{J} \cdot d\vec{a}$$ ← closed surface

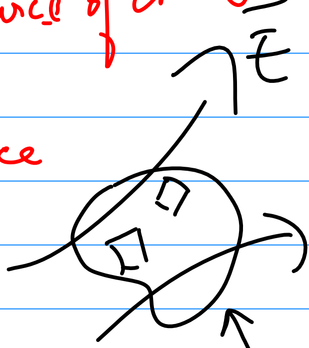
NET flow of charge IN/OUT of <u>closed</u> surface

If there is no source or sink of charges the net charge coming out of surface must be due to a change in charge within the surface

Cons. charge: $$\oint \vec{J} \cdot d\vec{a} = -\frac{dQ_{enclosed}}{dt} + \alpha_{source}$$

$\alpha = \phi$ no source of charge

Gauss's Law: $$\oint \vec{E} \cdot d\vec{a} = Q_{encl}/\epsilon_0$$ ← source of $\vec{E}$

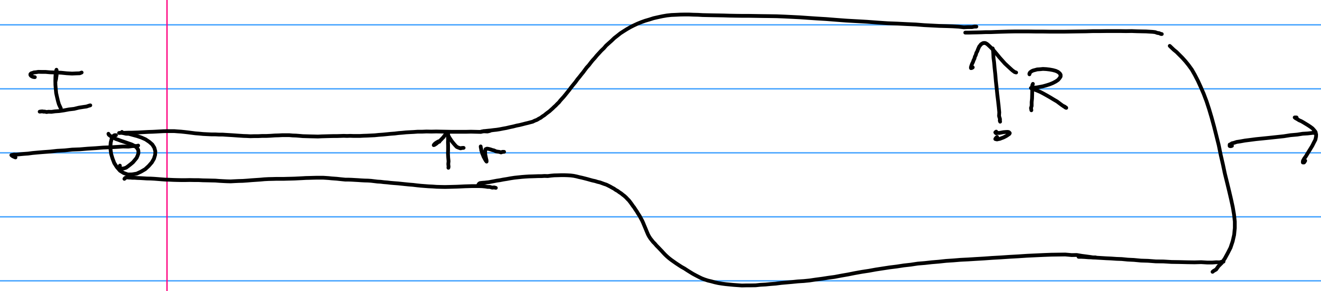


Is there a piling up of electric field lines in a closed surface? NO

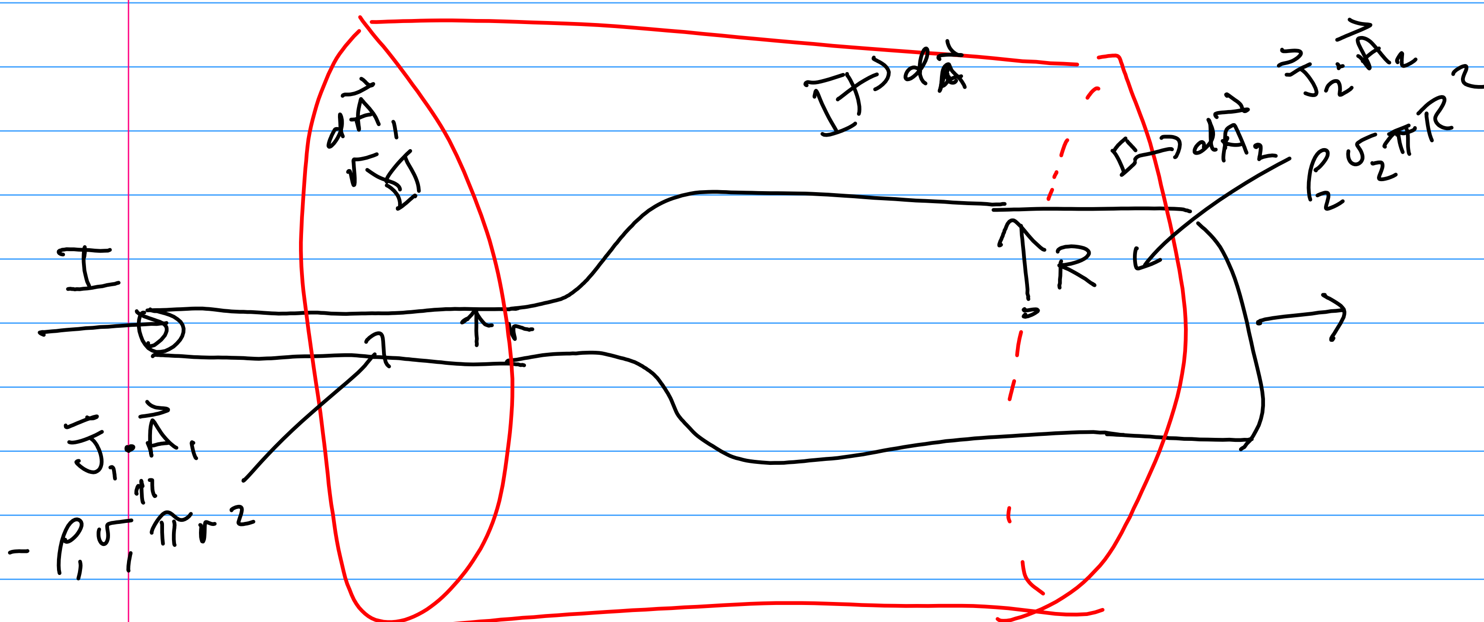
In a conductor $\rho \simeq$ constant

like an incompressible fluid

$$\int \vec{J} \cdot d\vec{a} = \int \rho \vec{v} \cdot d\vec{a} \quad \frac{\text{Coulombs}}{s} \text{ or Amps}$$



Informational: If the conductor is neutral what does the charge density mean?



$$\oint \vec{J} \cdot d\vec{a} = \frac{dQ_{\text{enclosed}}}{dt} = 0$$

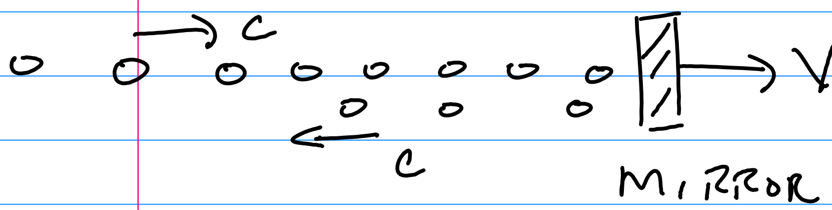
$$-\vec{J}_1 \cdot \vec{A}_1 + \vec{J}_2 \cdot \vec{A}_2 = 0 \; : \; \rho_1 v_1 \pi r^2 = \rho_2 v_2 \pi R^2$$

Congruous: How do I calculate the consequences of conservation of charge?

Informational: Does charge build up within the closed surface?

Example: conservation of photons in a laser beam reflecting from a moving mirror

Photons reflecting as a stream of particles 1-D



$$\vec{J}_{in} = \rho_{in}\vec{v}_{in} \rightarrow \lambda_{in} c$$

$$J_{reflected} = \rho_{ref}\vec{v}_{ref} \rightarrow \lambda_{ref} c$$

↓ more

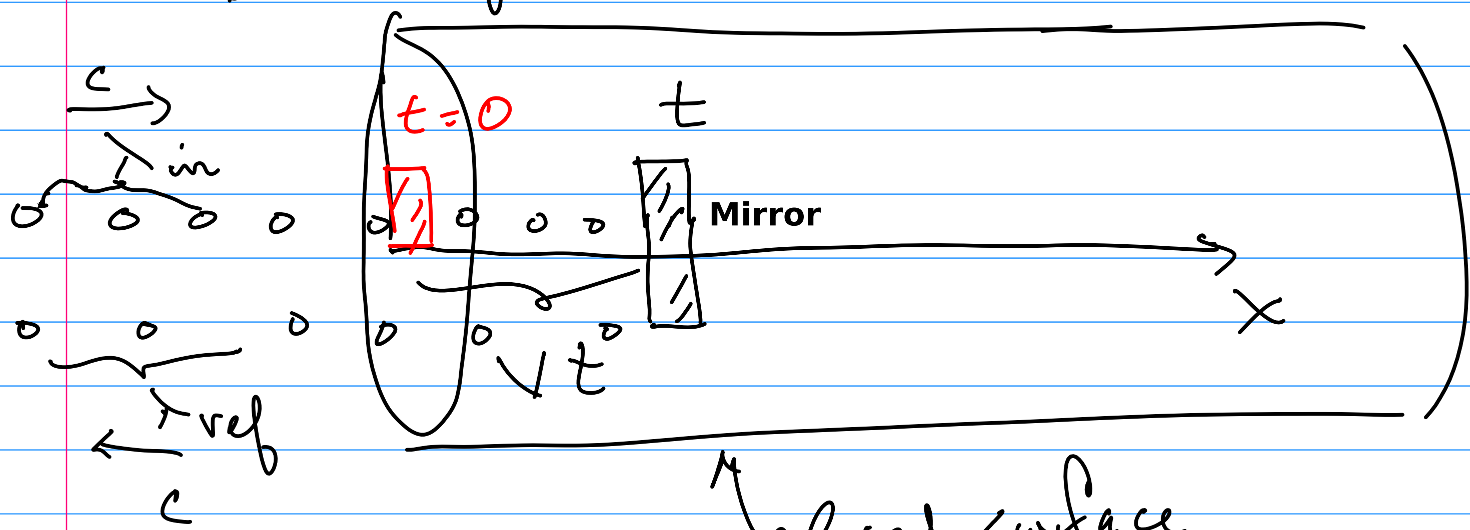
Model

$\vec{J}_{in} = \lambda_{in} c \hat{x}$

$\vec{J}_{ref} = -\lambda_{ref} c \hat{x}$

Reality

$10^{14} \frac{\text{photons}}{\text{sec}} \pm \sqrt{10^{14}}$

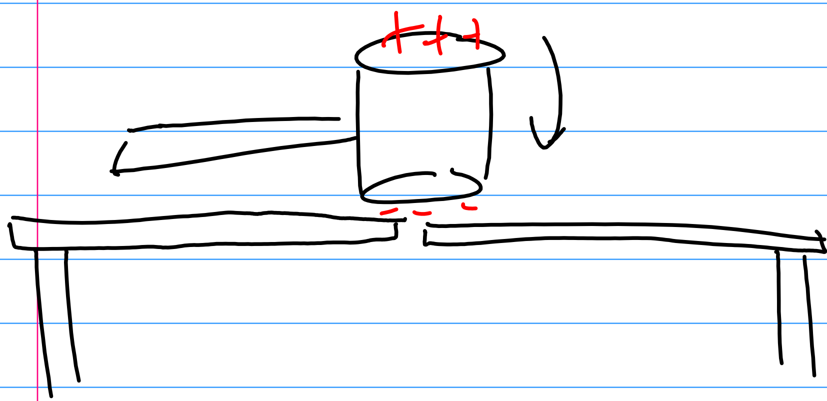


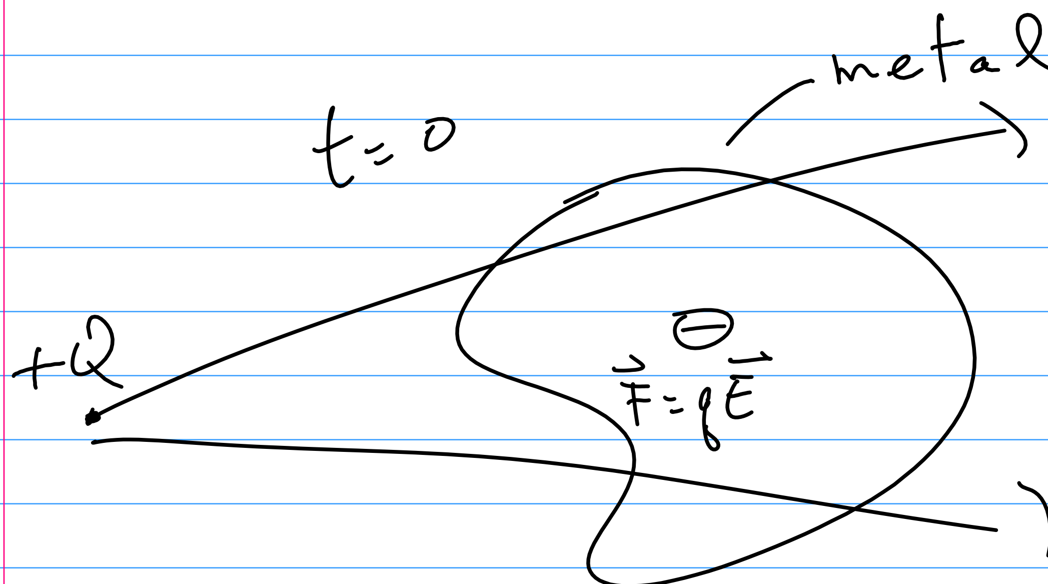
Conservation eqn ?

↓ more

(3) Conductors

Some electrons are free to move





Congruous: How are the charges distributed using Coulomb's law?