MATH225, Fall 2008 **Name:**
Worksheet 8 (4.3, Appendix B) **Section:**

For full credit, you must show all work and box answers.

1. A 1 kilogram mass is attached to a spring whose constant is 16 N/m, and the entire system is then submerged in a liquid that imparts a damping force such that the damping constant is 10.

   (a) Find the general solution for $y(t)$, the position of the mass at time t, when there is no external force.

   (b) Find the position of the mass at time t if the mass is released from 1 m below the equilibrium position ($y(0) = 1$) with no initial velocity, with no external force.

   (c) Find the position of the mass at time t if the mass is released 1 m below the equilibrium position ($y(0) = 1$) with an upward velocity of 12 m/s ($v(0) = -12$), with no external force.

   (d) Now add an external force, $f(t) = 2\sin(3t)$, to the harmonic oscillator. Find the general solution for the position of the mass at time t.

2. Consider the forced but undamped harmonic oscillator:
$y'' + y = 3\cos(\omega t), \quad y(0) = 0, \quad y'(0) = 0.$

(a) Find the particular solution for $\omega = \frac{9}{10}$. What phenomenon occurs at this value of ω?

(b) For ω from part (a), determine the frequency of beats and the frequency of rapid oscillations.

(c) For ω from part (a), how many rapid oscillations are there per beat?

(d) Find the particular solution for $\omega = 1$. What phenomenon occurs at this value of ω?

3. Using the power series method on $(1-t)\dfrac{dy}{dt} = y$,

(a) find the recurrence relation.

(b) Find the power series solution and then, using a known Taylor Series, write your answer in the form $y = f(t)$ (where $f(t)$ is not a series).

4. Using the power series method on $(1+t^2)y'' - 4ty' + 6y = 0$,

(a) find the recurrence relation.

(b) Find the power series solution and write your answer in the form $y = f(t)$ (where $f(t)$ is not a series).

(c) Solve for the unknown constants a_0 and a_1 using the initial condition $y(0) = 2$ and $y'(0) = -1$, and state the particular solution.