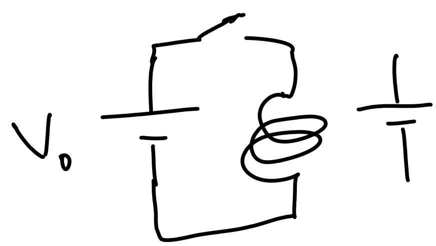
The learning objectives for exam 3 are that you will be able to:
-memorize Stokes theorem
-memorize Divergence theorem
-memorize differential and integral forms of conservation of charge
-understand Ohm's law and why the relaxation method can be used to determine the current density.
-know how to calculate the vector potential given a current distribution
-understand how to calculate the work required to assemble a current distribution in the many ways we discussed.
-be able to calculate the Faraday E due to a changing B using the integral form of Faraday's law.

You will be given the triangle diagrams and formulas related to energy in a charge distribution.

Homeorks will be outside my door.

Self inductance continued



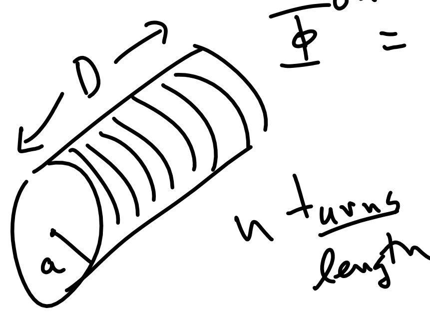
$$\mathcal{E}mf = -\frac{d\Phi_m}{dt}$$

The Faraday Emf for multiple loops is accounted for in the defn of inductance.

$$\Phi_m^{tot} = L i = N \Phi_m^{one\ turn}$$

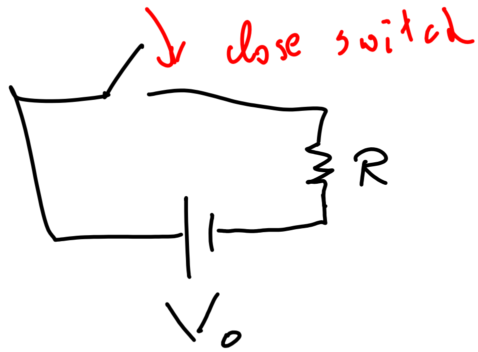
turns

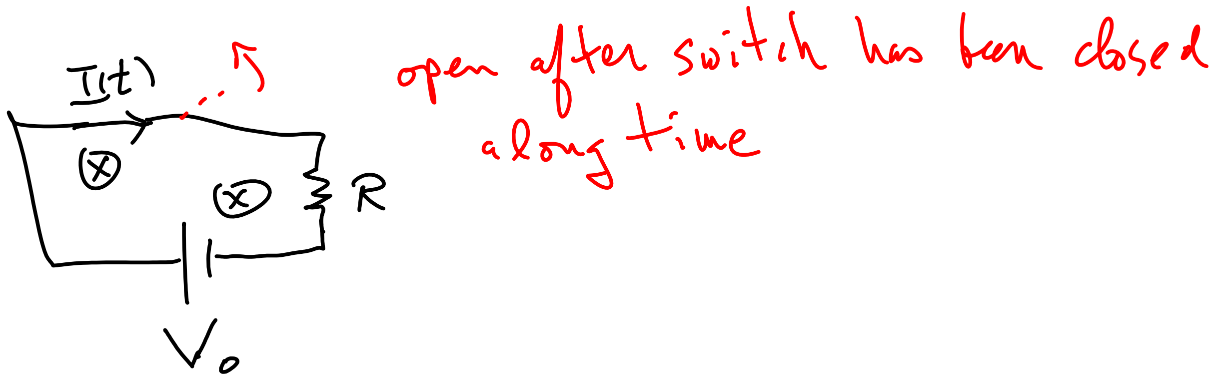
$$\mathcal{E}mf = -L\frac{di}{dt}$$



$$\Phi^{one} = \mu_0 n i \pi a^2$$

To calculate the self inductance of a solenoid use the defn.

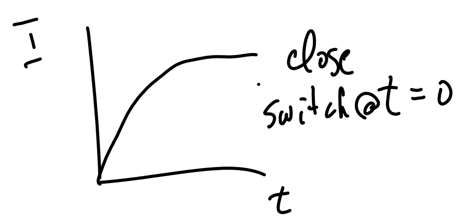




Closing the switch the initial current is zero due to the back emf from Faraday's law.

$$V_0 - IR - L\frac{dI}{dt} = 0$$

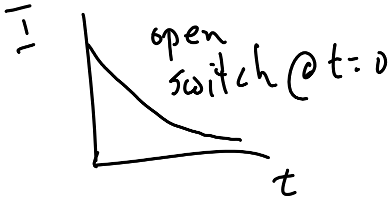
$$\left.\begin{array}{l} I = 0 \\ \dfrac{dI}{dt} = \dfrac{V_0}{L} \end{array}\right\} t = 0$$



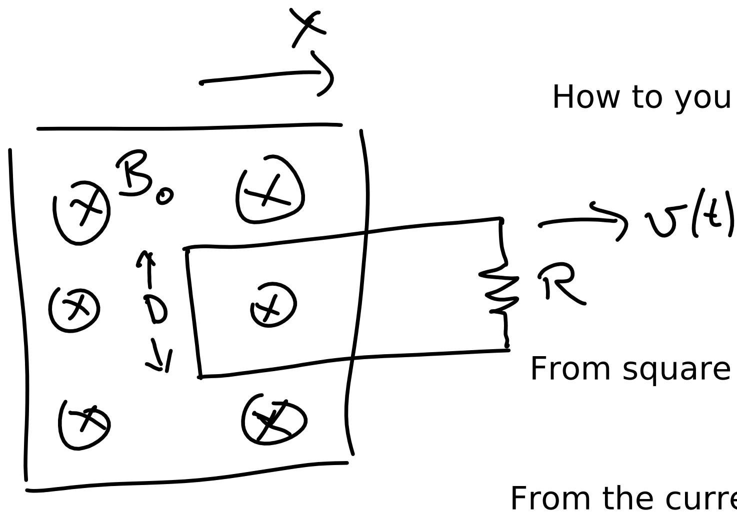
After the current is steady open the switch.

$$V_0 - IR - L\frac{dI}{dt} = 0$$

$$\left.\begin{aligned} I = I_0 = \frac{V_0}{R} \\ \frac{dI}{dt} = \frac{V_0 - IR}{L} \end{aligned}\right\} t = 0$$



Example of pulling a rectangular copper wire out of a magnetic field region.



How to you calculate the current in this circuit?

There are two magnetic fields: that from the square magnet and that from the current flowing in the rectangular wire.

From square magnet : $\Phi_m = \int \vec{B}_0 \cdot d\vec{a} = B\,Dx$

From the current in wire : $\Phi_{m_{self}} = LI$

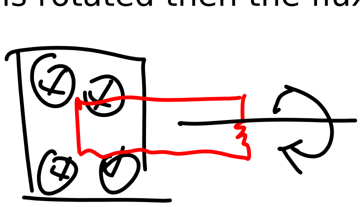
$$\frac{d\Phi_m^{total}}{dt} = \boxed{BDv - L\frac{dI}{dt} = IR}$$

How do you write Kirchoff's law?

$$\mathcal{E}mf = -\frac{d\Phi_m}{dt} \qquad \Phi_m = \int \vec{B} \cdot d\vec{a}$$

which B? B^{total}

If the wire is rotated then the flux changes due to a change in dot product angle.



$$\Phi_m = \int \vec{B} \cdot d\vec{a} = \int |\vec{B}||d\vec{a}| \cos(\Omega t)$$

The ways Faraday's law can generate an E:

1.) the area is changing

2.) the induced current generates an additional B which changes due to the changing current in the circuit. This B enclosed the whole circuit no just in the square.

3.) the angle in the dot product changes with time.

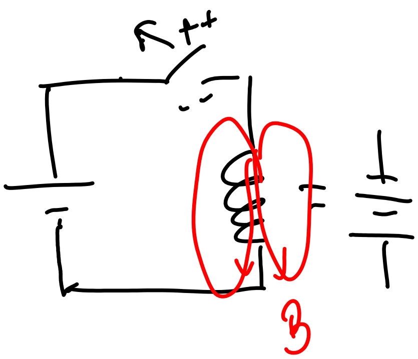
Energy stored in B

$$W_{me} = q\Delta V = qL\frac{dI}{dt}$$

$$\text{Power} = I\Delta V = IL\frac{dI}{dt} = \frac{dW_{me}}{dt}$$

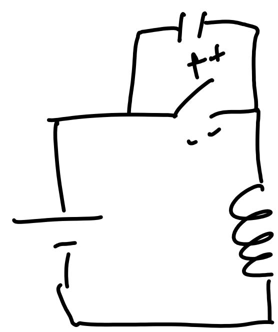
$$\text{Solve for } \int dW_{me} = \int IL\,dI = L\frac{I^2}{2}$$

open the switch after current has been flowing a Faraday emf is generated which tries to keep the current flowing. This can be large enough for a spark to be generated.



$$L\frac{dI}{dt} = \Delta V$$

This spark and the energy that was stored in the magnetic field can be transfered to a cap so no spark occurs



more

Express this energy in terms of the fields

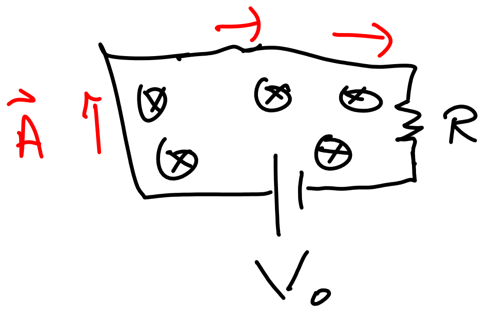
For homework fill out the steps in red

$$\Phi_m = \int \vec{B} \cdot d\vec{a} = \int \vec{A} \cdot d\vec{r} = LI$$

why?

Show using Stokes theorem

What does the line integral mean for this circuit? $\int \vec{A} \cdot d\vec{r}$



Using $W = \frac{1}{2} L I^2$

show

$$W = \frac{1}{2} I \oint \vec{A} \cdot d\vec{r}$$

bring inside

$$W = \frac{1}{2} \oint \vec{A} \cdot \vec{I}\, dr$$ where I points along $d\vec{r}$

Generalize this result to current density throughout a volume

$$W = \frac{1}{2}\int \vec{A} \cdot \vec{J}\, d\tau$$

Let's write this entirely in terms of the magnetic field using $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$$W = \frac{1}{2\mu_0}\int \vec{A} \cdot \vec{\nabla} \times \vec{B}\, d\tau$$

Integration by parts moves the curl from B to A using this identitiy

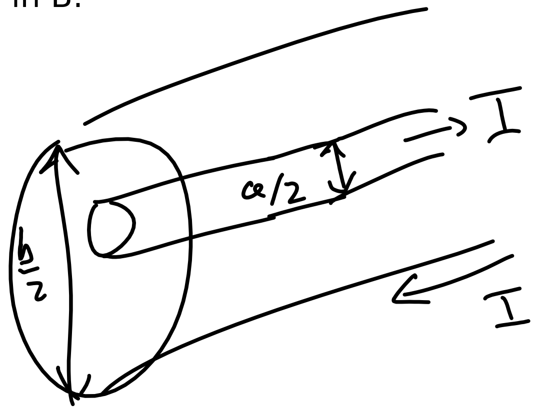
$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

Apply this identity and the divergence theorem to show that

$$W = \frac{1}{2\mu_0}\left[\int_{\text{vol}} B^2 d\tau - \oint_{\text{surface}} \vec{A} \times \vec{B} \cdot da \right]$$

Far from the currents the integrand of the second term goes to zero leaving the energy expressed only in terms of B.

Example of energy in B:

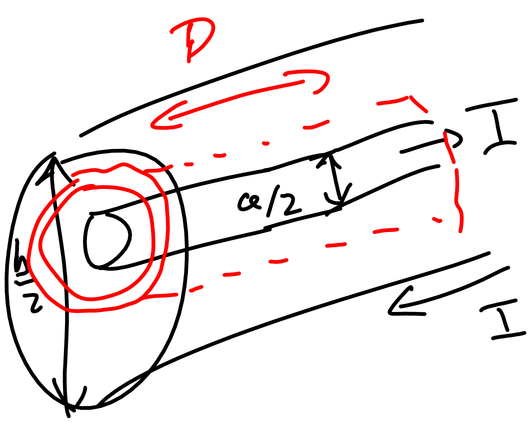


Ampere's law gives $B = \frac{\mu_0 I}{2\pi r}$ $\substack{a \\ b}$ inside & 0 outside

Energy per unit volume of the coaxial cable is

$$\frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2$$

Energy in the red shell of length D and thickness dr is



$$\frac{B^2}{2\mu_0} 2\pi r\, dr\, D$$

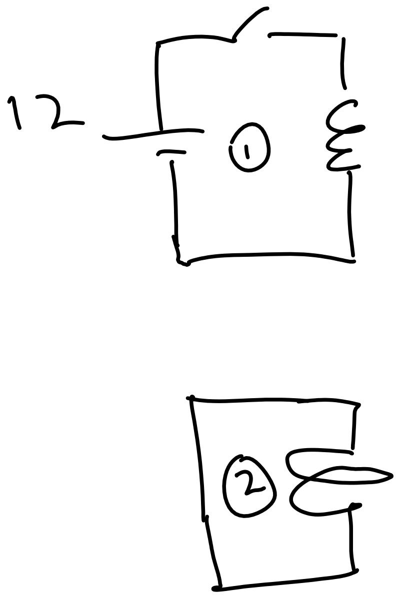
$$= \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi D r\, dr$$

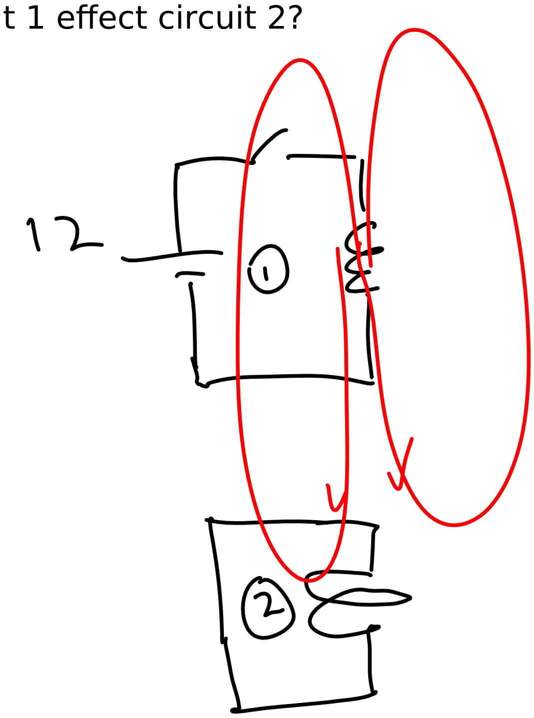
The total energy in length D is

$$\int_a^b \frac{\mu_0 I^2 D}{4\pi} \frac{dr}{r} = \frac{\mu_0 I^2 D}{4\pi} \ln \frac{b}{a} = \frac{1}{2} L I^2$$

Mutual inductance math

How does circuit 1 effect circuit 2?





(causal/creative) What fundamental law governs this behavior?

$$\Phi_m^{2\text{ due }1} \propto \; ?$$

$$\Phi_m^{2\text{ due }1} \propto I_1$$

$$\boxed{\Phi_m^{2\text{ due }1} \equiv M_{21} I_1}$$

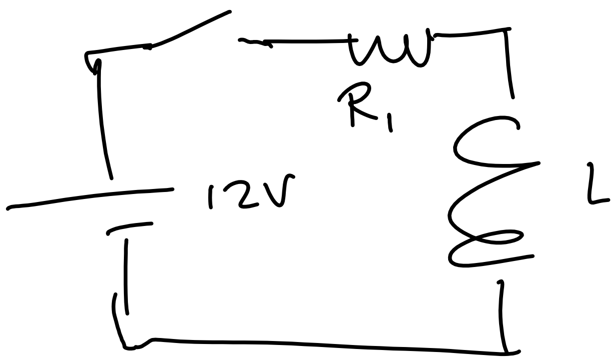
defn of M

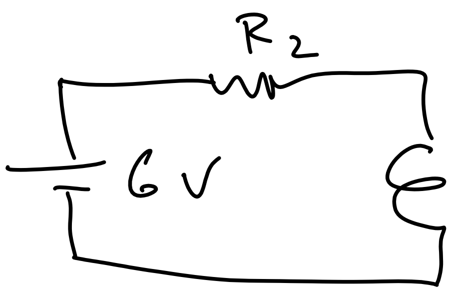
$$\oint \vec{E}_2 \cdot d\vec{r} = M_{21} \frac{dI_1}{dt}$$

assuming M_{21} constant

mutual inductance

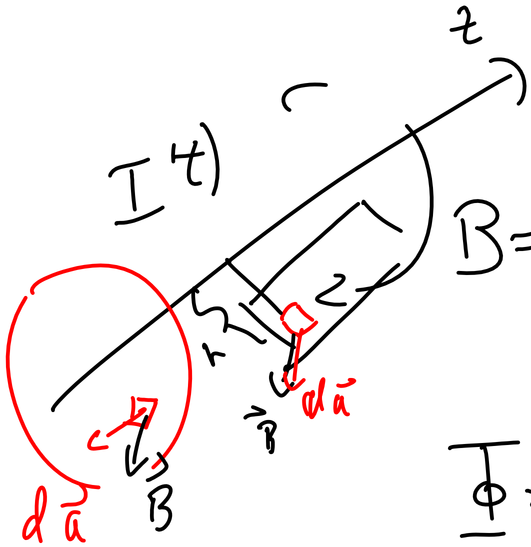
We can claculate L this way also.





$$12 - I_1 R_1 - L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} = 0$$

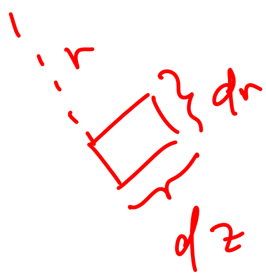
$$6 - I_2 R_2 - L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt} = 0$$



$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi_m}{dt}$$

$$B = \frac{\mu_0}{2\pi}\frac{I(t)}{r}$$

$$\Phi = \iint \frac{\mu_0}{2\pi} I(t)\,\frac{1}{r}\,dr\,dz$$



$$da = dr\,dz$$