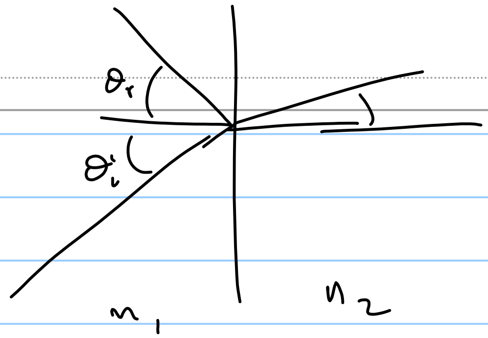
Note Title 7/17/2006



ME ⇒ wave eqn

Bndry conditions

$$(E_\perp)_i = (E_\perp)_{trans}$$

$$E_{\perp\, inc} + E_{\perp\, refl}$$

PDE & bndy — unique soln

$\lambda\nu = v_p = \frac{\omega}{k}$

EM waves in conductor

PDE $\quad \nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t}$ $\quad$ damped wave eqn

$\nabla^2 \to \frac{\partial^2}{\partial x^2}$

$\frac{d\omega}{dk} = v_g$

Dispersion relation: assume $E = E_0 e^{i(kx-\omega t)}$

$$\boxed{+k^2 = +\mu\epsilon\omega^2 + i\mu\sigma}$$

k is complex

$\frac{1}{\frac{dk}{d\omega}} = v_g$

$k = K_+ + iK_-$

$$e^{ikx} = e^{i(k_+ + ik_-)x} = e^{ik_+x}\,\underbrace{e^{-k_-x}}_{\text{damping}}$$

$d = \frac{1}{k_-}$ skin depth $\qquad k_+ = \frac{2\pi}{\lambda}$

$$v_{phase} = \frac{\omega}{k_+} = \frac{v_{\phi\,vac}}{n} = \frac{c}{n} \qquad n = \frac{c}{\omega}k_+$$

A is the area of the end cap

$D_{\perp,left}A - D_{\perp,right}A = \sigma A$

$d\vec{a}$ $\quad d\vec{a}$ $\quad \Delta x \to 0$ $\quad \}L$ $\quad \Delta x \to 0$

Reflection from a conducting surface

M.E.

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho_{free}} \Rightarrow D_{\perp,left} - D_{\perp,right} = \sigma$$

boundary

Gauss's Law

$$\epsilon_1(E_{\perp,inc} + E_{\perp,reflect}) - \epsilon_2 E_{\perp,trans} = \sigma$$

$e^{i(kx-\omega t)}$

$\vec{\nabla}\cdot\vec{B} = 0$

$$\int \vec{\nabla}\cdot\vec{D}\cdot da = \int \rho_f\, d\tau$$

$$\text{“}\oint \vec{D}\cdot d\vec{a} =$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t}$$

$$\int \vec{H}\cdot d\vec{l} = \int \vec{J}_f\cdot d\vec{a} + \frac{\partial\Phi_D}{\partial t}$$

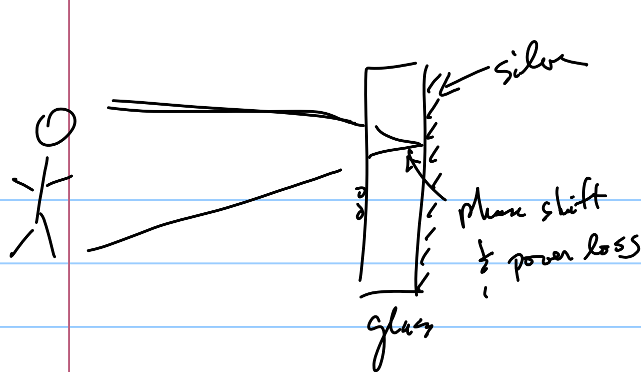
0 $\quad$ 0

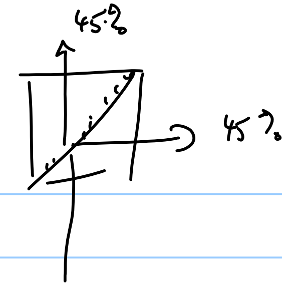
Stokes $\int \vec{\nabla}\times\vec{H}\cdot d\vec{a} = \int \vec{H}\cdot d\vec{l}$

$$\cancel{L}H_{\parallel,left} - \cancel{L}H_{\parallel,right} = 0$$

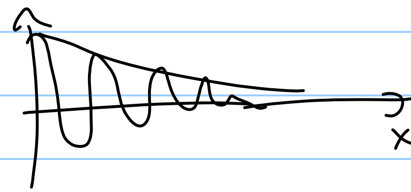
$$=$$

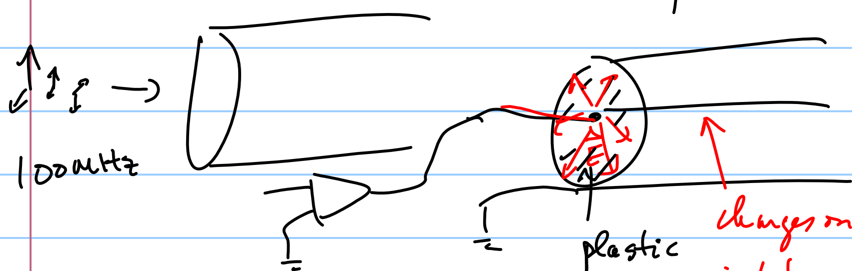
$$\frac{B_{\parallel,left}}{\mu_{left}} - \frac{B_{\parallel,right}}{\mu_{right}} = 0$$





EM waves in conductor

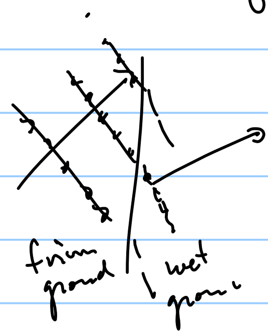


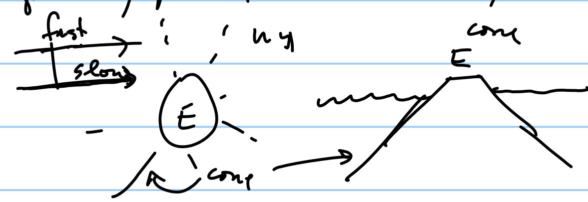


charges on surface & fields inbetween conductors that transmit signal

Exam review:

- EM waves on dielectric interface
- cons. laws: apply to simple case
- generation of EM wave: retarded potentials
- motion of waves (group vel, phase vel. bend)





Also want you to be able to calculate

- derivations in class
- problems

Review class notes, problems, text

EM waves in dielectrics: MICROSCOPIC THEORY

← non magnetic

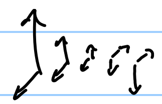
$$\vec\nabla\times\quad\left\{\vec\nabla\times\vec E = -\frac{\partial \vec B}{\partial t} = -\mu_0\frac{\partial H}{\partial t}\right\}\qquad \vec D = \epsilon\vec E + \vec P$$

$$\frac{\partial}{\partial t}\quad\left\{\vec\nabla\times\vec H = \vec J + \frac{\partial \vec D}{\partial t} = \vec J + \epsilon_0\frac{\partial \vec E}{\partial t} + \frac{\partial \vec P}{\partial t}\right\}$$

$$\vec\nabla\times\vec\nabla\times\vec E = \vec\nabla(\vec\nabla\cdot\vec E) - \nabla^2\vec E = \frac{1}{c^2}\frac{\partial^2 \vec E}{\partial t^2} + \mu_0\frac{\partial^2 \vec P}{\partial t^2} + \mu_0\underbrace{\frac{\partial \vec J}{\partial t}}_{0}$$

Two cases $\quad \vec J = 0 \quad$ dielectric

EM wave is going thru matter which consists of atoms



$\hat y$

$$\vec F = q\vec E + q\vec v\times\vec B = qE + qv\frac{E}{c} = qE\Big(1+\underbrace{\frac{v}{c}}_{\approx 0}\Big)$$

$E = v_{phase}B = cB$

$\vec P$ $\frac{\text{dipole mom}}{\text{vol}}$

$$\frac{\text{atoms}}{\text{vol}}\ \underbrace{\frac{\text{dipole mom}}{\text{atom}}}_{qy} = \vec P = Nq\hat y$$

N: # of atom / vol

$\hat y$: ← complex displacement of electron from proton

Newton's 2^{nd}

$$qE - m\gamma \frac{d\tilde{y}}{dt} - Kr = m\frac{d^2\tilde{y}}{dt} \qquad \text{ODE}$$

$E \;\; '' \;\; E_0 e^{-i\omega t}$ ↑ damping ↑ spring

assume $\tilde{y}(t) = \hat{y}_0 e^{-i\omega t}$

↑ phase shift

plug in ODE

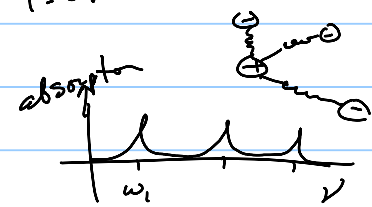
density of atoms

$$P = \frac{Ne^2/m}{\omega_0^2 - \omega^2 - i\omega\gamma} E$$

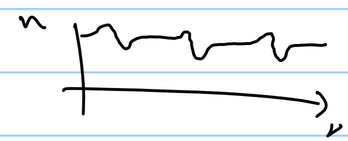
goes into wave eqn

PDE

↑ resonant freq



get dispersion relation by assuming



Dispersion relation: assume $E = E_0 e^{i(kx - \omega t)}$

complex $k \Rightarrow$ absorption & new $\lambda \rightarrow \frac{\lambda_{vac}}{n}$